請寫下計算過程或思考步驟

1　(8%+8%)　Consider an attractive force field, given by $\boxed{\vec{F} = -k\vec{r}}$, observing the Hooke's law.　Find the work done against the force, i.e., evaluating $\boxed{-\int_{(1,2,3)}^{(2,1,3)} \vec{F} \cdot d\vec{r}}$, when going from (1,2,3) to (2,1,3) via the following paths:

(a)　along the straight line connecting (1,2,3) and (2,1,3).

(b)　along a circle of radius $\sqrt{5}$ and centered at (0,0,3).

2　(8%+8%)　Consider an ordinary differential equation (ODE) given by $\boxed{x\dfrac{dy}{dx} - y + x = 0}$.

(a)　Find the integration factor $\mu(x,y)$ to turn this ODE into an exact differential equation. [Hint: Try $\mu(x,y) = x^{\alpha} y^{\beta}$.]

(b)　Solve this differential equation.

3　(10%+10%+10%)

(a)　Obtain the general solution of the homogeneous ODE $\boxed{y'' - 2y' + y = 0}$.

(b)　Find a particular solution of the inhomogeneous ODE $\boxed{y'' - 2y' + y = xe^x}$.

(c)　Find the solution of $\boxed{y'' - 2y' + y = xe^x}$ satisfying the initial conditions: $y(x=0) = 0$ and $y'(x=0) = 1$.

4　(4%+12%+6%+16%)　Consider a periodic function of period 2, defined as $f(x) = |x|$ in the range $-1 \le x \le 1$.

(a)　Sketch the function $f(x)$.

(b)　Find the sine/cosine Fourier series for the function $f(x)$.

(c)　Using the Fourier expansion obtained for $f(x)$ in (b), find the sine/cosine Fourier series for the function $g(x)$ with the period of 2 defined as 0 for $-1 \le x < 0$ and 1 for $0 \le x < 1$.

(d)　Use the Fourier series in (b) and (c) to show that

$$S_1 = 1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \cdots = \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} = \frac{\pi^2}{8} \quad \text{and}$$

$$S_2 = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2n-1} = \frac{\pi}{4}.$$