淡江大學 107 學年度碩士班招生考試試題

系別：數學學系　　　　　　科目：線性代數　　　　26 - 1

考試日期:3 月 11 日(星期日) 第 2 節　　　　本試題共 7 大題，　1 頁

SHOW ALL YOUR WORK. NO CREDIT WILL BE GIVEN FOR ONLY SOLUTION.

1. Let $A=\begin{bmatrix} 1 & -2 & 3 & -2 & 1 \\ -3 & 6 & 1 & 0 & 1 \\ 2 & -4 & -4 & 2 & -2 \end{bmatrix}$

   Find the rank and nullity of A. Also find the basis of the column space, row space and null space of A. (20 %)

2. Define an inner product on R^4 by $<v,w>=x_1y_1+2x_2y_2+x_3y_3+4x_4y_4$ for all $v=(x_1,\ x_2,\ x_3,\ x_4)$,

   $w=(y_1,\ y_2,\ y_3,\ y_4)$ in R^4

   Let $B=\{(1,2,-1,0),\ (0,3,2,1),\ (0,1,0,1)\}$, Use the Gram-Schmidt process to find an orthogonal basis of span(B). (12 %)

3. Let $T:P_2\to P_2$ be a linear transformation and $B=\{1,\ x,\ x^2\}$ be a basis of P_2. If the matrix representation of T relative to basis B is $M_B=\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 1 \end{bmatrix}$. Find the eigenvalues and eigenvectors of T (12%)

4. Let $A=\begin{bmatrix} 1 & 5 & -3 & 3 \\ -4 & -8 & 4 & -4 \\ 2 & 4 & -4 & 2 \\ 5 & 8 & -5 & 3 \end{bmatrix}$. Find matrices P and J so that $J=P^{-1}AP$ is the Jordan canonical form for A.

   Hint: -2 is an eigenvalue of A with multiplicity 4. (20 %)

5. Let $V=M_{n,n}$, the set of all n by n matrix. Let $U=\{A;\ \ A^T=A\}\subset V$ and $W=\{A;\ \ A^T=-A\}\subset V$, show that $V=U\oplus W$. (12 %)

6. If A is diagonalizable and $\lambda\geq 0$ for each eigenvalue of A, show that $A=B^2$ for some matrix B (12 %)

7. Let A be an n by n real symmetric matrix, Let $\lambda_1,\lambda_2,\lambda_3$ be distinct eigenvalues of A with eigenvectors v_1,v_2,v_3, Show that $\{v_1,\ v_2,\ v_3\}$ is linear independent and orthogonal. (12 %)