

淡江大學九十三年學年度碩士班招生考試試題

系別：數學學系

科目：微積分 60%及線性代數 40%

准帶項目請打「○」否則打「×」

簡單型計算機

本試題共 / 頁

請詳列演算過程, 否則不予計分, 每題 10 分, 共 10 題

- Determine whether the series, $\sum_{n=1}^{\infty} \frac{n \cos n\pi}{1+n^2}$, is convergent or divergent?
- Determine whether the improper integral $\int_c^{\infty} (\ln x)^{-\frac{1}{2}} \frac{1}{x} dx$ is convergent or divergent?
- Evaluate the given integral $\int \sin(\ln x) dx$.
- Use power series to approximate the value of $\int_0^1 \frac{\sin x}{x} dx$ accurate to four decimal place.
- Evaluate the given double integral $\int_0^1 \int_1^1 x e^{y^3} dy dx$.
- Find the minimum value of the function $f(x, y, z) = 4xy + 6xz + 6yz$ subject to the constraint $xyz = 144$.
- Compute the rank of $A = \begin{bmatrix} 1 & 1 & 2 & 3 \\ 2 & 4 & 1 & 0 \\ 1 & 5 & -4 & -9 \end{bmatrix}$ and find bases for the row space and the column space of A.
- Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix}$. Find a matrix P such that $P^{-1}AP$ is a diagonal or upper triangular matrix.
- Let P_3 denote the set of polynomials with degree less or equal to 3. Let $\langle p, q \rangle = \int_{-1}^1 p(x)q(x)dx$, where $p(x), q(x)$ are polynomials, be a inner product on P_3 . Find an orthogonal basis of P_3 .
- Let A be a real n by n matrix and b be a real n by 1 vector. Show that the equation $Ax = b$ has solution if and only if $y'b = 0$ for all real n by 1 vector y satisfying $A'y = 0$. Here t denotes the transpose of the given objects.