

# 淡江大學八十九學年度碩士班招生考試試題

系別：數學學系

科目：微積分 60%及線性代數 40%

本試題共 1 頁

1. (5%) Evaluate

$$\lim_{n \rightarrow \infty} \sum_{k=0}^{n-1} \frac{1}{\sqrt{n^2 + k^2}}$$

2. (10%) Let  $\lim_{n \rightarrow \infty} a_n = \alpha$ . Prove that

$$\lim_{n \rightarrow \infty} \frac{a_1 + a_2 + \dots + a_n}{n} = \alpha.$$

3. (10%) Find

(a)

$$\lim_{h \rightarrow 0} \frac{1}{h} \int_x^{x+h} \frac{du}{\sqrt{u^3 + 1} - u}$$

(b)

$$\lim_{(x,y) \rightarrow (0,0)} (x^2 + y^2) \ln(x^2 + y^2).$$

4. (10%) Find the interval of convergence of

$$\sum_{n=1}^{\infty} \frac{(n!)^3}{(3n)!} x^{3n}.$$

5. (10%) Evaluate

$$\int \frac{d\theta}{2 + 3\cos\theta}$$

6. (5%) Let

$$f(x) = \begin{cases} e^{-\frac{1}{x^2}} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

Show that

(a) Show that  $f'(0) = 0$ .

(b) Show that  $f''(0) = 0$ .

7. (10%) Find the volume of the ellipsoid  $x^2 + y^2 + 4z^2 = 4$ .

8. (5%)

$$A = \begin{bmatrix} 1 & 3 & 1 & 3 \\ 0 & 1 & 1 & 0 \\ -3 & 0 & 6 & -1 \\ 3 & 4 & -2 & 1 \\ 2 & 0 & -4 & -2 \end{bmatrix}.$$

Find a basis for the column space of the matrix  $A$

9. (5%) Find the solution to the least squares problem

$$\begin{bmatrix} 1 & 1 \\ 1 & 2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} c_0 \\ c_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix}.$$

10. (10%) Let  $A$  be an  $n \times n$  matrix. Suppose  $\lambda_1, \lambda_2, \dots, \lambda_k$  are distinct eigenvalues of  $A$  and  $\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_k$  are the associated eigenvectors. Then  $\{\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_k\}$  is linearly independent.

11. (10%)

$$A = \begin{bmatrix} 1 & -1 & -1 \\ 1 & 3 & 1 \\ -3 & 1 & -1 \end{bmatrix}.$$

Evaluate  $A^{17}$ .

12. (10%) If  $A$  is a symmetric matrix, then its eigenvalues are real numbers.