

淡江大學 95 學年度碩士班招生考試試題

70

系別：數學學系

科目：高等微積分

准帶項目請打「V」	
X	簡單型計算機

本試題共 1 頁

- Let  $A$  be a subset of  $\mathbb{R}$  (the set of real numbers) and  $F: \mathbb{R} \rightarrow \mathbb{R}$ .  
State or explain the following theorem or terminology.
  - $A$  is sequentially compact. (2 points)
  - $F$  is uniformly continuous on  $\mathbb{R}$ . (2 points)
  - Point  $p$  is a limit point of  $A$ . (2 points)
- Let  $f: \mathbb{R}^2 \rightarrow \mathbb{R}$  be a differentiable function and there is a natural number  $n$  such that  $f(tx, ty) = t^n f(x, y)$  for all real numbers  $t, x$  and  $y$ . Prove that  $x \frac{\partial f}{\partial x}(x, y) + y \frac{\partial f}{\partial y}(x, y) = n f(x, y)$ . (10 points)
- Let  $\{a_n\}$  and  $\{b_n\}$  be bounded sequences in  $\mathbb{R}$ .
  - Write the definition of  $\limsup_{n \rightarrow \infty} \{a_n\}$  and  $\liminf_{n \rightarrow \infty} \{a_n\}$ . (4 points)
  - Prove that  $\limsup_{n \rightarrow \infty} \{a_n + b_n\} \leq \limsup_{n \rightarrow \infty} \{a_n\} + \limsup_{n \rightarrow \infty} \{b_n\}$  (10 points)
- Show that  $\lim_{(x,y) \rightarrow (0,0)} \frac{2yx^2}{x^4 + y^2}$  does not exist. (10 points)
- Let  $z = f(x, y)$  be a differentiable function,  $f(1, 1) = 1$  and  $z^3 - xy + yz + y^3 - 2 = 0$ . Find  $\frac{\partial z}{\partial x}$  at  $(1, 1)$ . (10 points)
- Prove that if a sequence of continuous function on  $\mathbb{R}$  converges uniformly, then the limit function  $f$  is continuous. (10 points)
- Prove that if  $f$  is a continuous real-valued function on  $[a, b]$ , then  $f$  is uniformly continuous on  $[a, b]$ . (10 points)
- Prove that if  $\sum_{n=1}^{\infty} a_n$  converges absolutely, then  $\sum_{n=1}^{\infty} a_n \sin nx$  converges uniformly on  $\mathbb{R}$ . (10 points).
- Find the radius of convergence and interval of convergence of the series  $\sum_{n=1}^{\infty} (-2)^n \frac{x^n}{\sqrt{n}}$ . (10 points)
- Find  $\frac{dy}{dx}$  if  $y = \int_0^{\sin x} \sqrt{1+t+t^2} dt$ . (10 points)