淡江大學八十八學年度日間部轉學生入學考試試題

系別：理工組二年級　　科目：微積分

本試題共 2 頁

本試題雙面印製

※注意：第一題為填充題，請在答案卷第一頁依序寫上題號再寫答案，不必寫出演算過程。
第二、三、四、五、六為計算證明題，務必要有演算過程。

一、填充題（共10小題，每小題5分）

1. Find the area of the loop of the graph of the equation $r\cos\theta = \cos 2\theta$.

2. Evaluate $\iiint_S \frac{|z|}{(x^2+y^2+z^2+1)^{\frac{1}{2}}} dV$, where S is the ball $x^2+y^2+z^2 \le 1$.

3. Let $f(x) = \int_0^{\pi} \frac{y}{x^2}\cos\frac{y}{x}\,dy$. Then $\int_{\frac{1}{2}}^{2} f(x)\,dx = ?$

4. Let $f(x) = \int_0^x \frac{\ln(1+t)}{t}\,dt$. Find $f'''(0) = ?$

5. Find $\lim_{x\to 0}\left(\frac{1}{x\sin^{-1}x} - \frac{1}{x^2}\right)$.

6. Let $f(x) = (x^2+1)^7(x^3+2)^5/(x^4+1)$. Then $f'(1) = ?$

7. Let $f(x,y) = xy(2x-y)/(|x|+|y|)$ If $(x,y)\neq(0,0)$, $f(0,0)=0$
then $\frac{\partial f}{\partial x}(0,y) = ?$

8. Find the length of the curve $x = \frac{y^4}{16} + \frac{1}{2y^2}$, $-2 \le y \le -1$.

9. Given the parametric curve $x = \theta - \sin\theta$, $y = 1-\cos\theta$.
Find $\frac{d^2y}{dx^2}$ at $\theta = \frac{\pi}{3}$.

10. Let $y = f(x)$ be a differentiable function on $(-\infty,\infty)$.
If $-2x+1+3y^2\frac{dy}{dx} = 0$ and $f(2) = 2$, then $f(x) = ?$

◀注意背面尚有試題▶

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(10分)二. Let f be a differentrable function on $(0, \infty)$ and $z = f(\frac{y}{x})/x$. Show that

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} + z = 0.$$

(10分) 三. Find and classify the critical points of

$$f(x, y) = x^3 - 12xy + 8y^3 + 1.$$

(10分) 四. Find the interval of convergence for the power series

$$\sum_{n=0}^{\infty} \frac{(2^n+1)(-1)^n}{3^n(2n+1)} x^n.$$

(10分) 五. Show that if $0 < \alpha < \beta < \frac{\pi}{2}$, then

$$\alpha \sin\beta\cos\alpha > \beta \sin\alpha\cos\beta.$$

(10分) 六. Determine whether the integral $\int_e^{\infty} \frac{1}{(\ln x)^{3/2}\sqrt{x^2+4}}\,dx$ is convergent or divergent.

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