請盡量詳細寫出各解題步驟及計算過程，否則不予計分。

1. (25%) Please find (1) $\dfrac{\partial}{\partial y}\left[\sin(x^2+y^2)\,e^{-(x+y)^2}\right]$; (2) $\displaystyle\int_0^\infty u\,e^{-u}\left(1+e^{-u}\right)du$.

2. Given a matrix $A=\begin{pmatrix} 2 & 0 & -1 \\ 0 & -3 & 0 \\ -1 & 0 & -2 \end{pmatrix}$,

   (1) (20%) find its eigenvalues and normalized eigenvectors;

   (2) (5%) show that the corresponding eigenvectors are mutually orthogonal.

3. The differential equation is given by $\dfrac{d^2 f}{dt^2} - 4\dfrac{df}{dt} + 3f = 0$.

   (1) (18%) Find the general solution of the equation.

   (2) (7%) When $f(0)=0$ and $\dfrac{df}{dt}=1$ at $t=0$, determine the constants in the general solution of the equation.

4. (25%) A vector fields is given by $\mathbf{V}=(xy^2+z)\mathbf{i}+(x^2y+2)\mathbf{j}+x\mathbf{k}$, where $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are unit vectors in Cartesian coordinate system.

   (1) (5%) Calculate $\nabla\cdot\mathbf{V}$ and $\nabla\times\mathbf{V}$.

   (2) (5%) Is $\mathbf{V}$ a conservative field? Why?

   (3) (8%) Evaluate directly the line integrals $I=\displaystyle\int_A^B \mathbf{V}\cdot d\mathbf{r}$ along the path given by $x=t$, $y=2t$ and $z=4t^2$, where A and B are the points (1, 2, 4) and (3, 6, 36). Hint: at A, $t_A=1$ and at B, $t_B=3$.

   (4) (7%) Find the potential function of the conservative field, i.e., find a scalar function ϕ to make $\mathbf{V}=\nabla\phi$.