淡江大學 109 學年度日間部寒假轉學生招生考試試題

系別：數學系資統組三年級　　　科目：機率與統計學　11-1　11

考試日期：1 月 18 日(星期一) 第 1 節　　　本試題共 7 大題， 1 頁

1.(10%) If $P(A) = 0.3, P(A \mid B) = 0.3$ and $P(B) = 0.5$. Compute $P(B \mid A)$ and $P(A \mid B^c)$ where B^c is the complement of B.

2.(10%) Let $X_1, X_2, \cdots, X_n$ be a random sample of size n with p.d.f. $f(x) = \theta x^{\theta-1}, 0 < x < 1, 0 < \theta < \infty$. Find the mle (maximum likelihood estimate) of θ.

3.(20%) The life time T (in years) of a light bulb has the exponential distribution $f(t) = \frac{1}{2}e^{-t/2}$, $t > 0$.

(i) What are the mean $(E(T))$ and variance $(Var(T))$ of the life time of the light bulb.

(ii) If the light bulb has been used for one year, what is the probability that this bulb can be used at least for one more year $(P(T \geq 2 \mid T \geq 1))$.

4.(15%) Let X and Y have the joint p.m.f. $f(x, y) = \frac{x+y}{21}, x = 1, 2, 3, y = 1, 2$. Compute $E(X \mid Y = 2)$ and $Var(X \mid Y = 2)$.

5.(15%) Consider the simple linear regression $Y_i = \alpha + \beta X_i + \epsilon_i,\ i = 1, \cdots, n$, where ϵ_i are i.i.d. $N(0, \sigma^2)$, let $\hat{\alpha}$, $\hat{\beta}$ be the least squares estimates of α and β.

(i) Prove that $\hat{\beta} = S_{XY}/S_{XX},\ \hat{\alpha} = \bar{Y} - \hat{\beta}\bar{X}$, where $S_{XY} = \sum_{i=1}^{n}[(Y_i - \bar{Y})(X_i - \bar{X})]$, and $S_{XX} = \sum_{i=1}^{n}(X_i - \bar{X})^2$.

(ii) Show that $\hat{\beta}$ is unbiased.

6.(10%) Supposr that $W_1, \cdots, W_{10}$ are iid continuous random variables. Denote the order statistics by $W_{(1)} < W_{(2)} < \cdots < W_{(10)}$. Let ξ denote the 60% percentile of the distribution, that is $P(W_i \leq \xi) = 0.6$. What is the confidence level of the confidence interval $(W_{(5)}, W_{(8)})$ for estimating ξ.

7.(20%) Let X_1, X_2 and X_3 be i.i.d. random variables from $N(\mu, 12)$. Consider the test that has null hypothesis $H_0 : \mu = 0$ v.s. alternative hypothesis $H_1 : \mu = 1$.
i). Construct a critical region C so that the significence level is 0.05.
ii). Find the power of the above test.
iii). Find a confidence lower bound $L(X_1, X_2, X_3)$ such that $P(L(X_1, X_2, X_3) \leq \mu) = 0.95$.
v). Compute the p-value if we observe $X_1 = 1, X_2 = 0.96$ and $X_3 = 1.96$.