淡江大學109學年度日間部寒假轉學生招生考試試題

系別：數學系數學組三年級　　　　科目：線性代數　　　　10-10

考試日期：1月18日(星期一) 第1節　　　　本試題共　5　大題，1　頁

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1. Let $A=\begin{bmatrix}1 & -2 & 0 & 2 & 1 & 7\\ 2 & -4 & 0 & 4 & 0 & 2\\ 0 & 0 & 1 & 5 & 0 & -3\\ 1 & -2 & 1 & 7 & 0 & 2\end{bmatrix}$, find a basis of the column space, row space of A and a basis of the null space of A. (20 %)

2. Let $A=\begin{bmatrix}3 & 2 & -1\\ 2 & 0 & 2\\ -1 & 2 & 3\end{bmatrix}$. Find an orthogonal matrix P and a diagonal matrix D, such that $P^{-1}AP=D$. (20 %)

3. Let $B_1=\left\{\begin{bmatrix}1\\1\end{bmatrix}, \begin{bmatrix}1\\-1\end{bmatrix}\right\}$ be a basis of R^2, $B_2=\{1, x, x^2, x^3\}$ be a basis of $P_3(R)$, set of polynomials with degree less or equal to 3. Let $T: R^2 \to P_3(R)$ be a linear transformation with matrix representation relative to B_1, B_2 is $\begin{bmatrix}1 & 0\\ 0 & 2\\ -1 & 0\\ 3 & -1\end{bmatrix}$. Let $S: R^2 \to P_3(R)$ be a linear transformation defined by $S(\begin{bmatrix}a\\b\end{bmatrix})=a+bx+(a+b)x^2+(a-b)x^3$

a) Find the matrix representation of S relative to the bases B_1, B_2. (10 %)

b) Find $(3T-2S)(\begin{bmatrix}3\\5\end{bmatrix})$. (10 %)

4. Let $W=\left\{\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix}: \ x_1+2x_2=0, \ \ x_2-x_3+2x_4=0\right\}$, find a basis of W and $W^{\perp}$. (20 %)

5. Let U, W be nonempty subspaces of a vector space V. Suppose that $U \neq \{O\}$, $W \neq \{O\}$, here O is the zero element of V and $U \cap W=\{O\}$. Show that if $u \in U$, $w \in W$, and they are nonzero elements, then u, w are linear independent. (20 %)