淡江大學 109 學年度日間部轉學生招生考試試題

系別：數學系數學組三年級　　科目：線性代數　　22-1

考試日期：7 月22日 (星期三) 第 1 節　　本試題共 7 大題，共 1 頁

請詳列計算過程，否則不予計分。

1. Let $A = \begin{bmatrix} -2 & 0 & -1 \\ 0 & 2 & 0 \\ 3 & 0 & 2 \end{bmatrix}$.

   (1) Find the characteristic polynomial of A. (10%)

   (2) Find all eigenvalues and eigenvectors of A. (10%)

   (3) Find an invertible matrix P such that $P^{-1}AP = D$ is a diagonal matrix. (10%)

2. Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \\ 2 & 3 & 1 \end{bmatrix}$. Find the inverse matrix A^{-1} of A. (10%)

3. Let $A = \begin{bmatrix} 3 & 2 & 1 & 1 & 4 \\ 2 & 3 & 4 & 0 & 5 \\ 1 & 1 & 1 & 1 & 5 \\ 4 & 2 & 0 & 1 & 2 \end{bmatrix}$.

   (1) Find the reduced row echelon form, rank, and nullity of A. (5%)

   (2) Find a basis of the null space of A. (5%)

   (3) Find a basis of the row space of A. (5%)

4. Let V be the vector space of all $n \times n$ matrices. Given two $n \times n$ matrices A and B, let $W = \{X \in V : AXB = X\}$. Prove that W is a subspace of V. (10%)

5. Let $P_2 = \{a_0 + a_1 x + a_2 x^2 : a_i \in \mathbb{R}\}$ be the vector space of polynomials of degree at most 2, let $T : P_2 \to P_2$ be defined by $T(p(x)) = p(x-1)$.

   (1) Prove that T is a linear transformation. (5%)

   (2) Find the matrix for T with respect to the ordered basis $\{1, x, x^2\}$. (10%)

6. Let V and W be finite dimensional vector spaces, and $T : V \to W$ be a linear transformation. Prove that if T is one-to-one, then $\dim(V) \leq \dim(W)$. (10%)

7. Let A be an $n \times n$ matrix, and $\mathbf{v}_1, \ldots, \mathbf{v}_k$ be eigenvectors of A corresponding to distinct eigenvalues $\lambda_1, \ldots, \lambda_k$, respectively. Prove that $\{\mathbf{v}_1, \ldots, \mathbf{v}_k\}$ is a linearly independent set. (10%)