淡江大學 109 學年度進修學士班轉學生招生考試試題

系別：工組二年級　　科目：微積分　　4-1

考試日期：7 月 23 日（星期四）第 2 節　　本試題共 7 大題，共 1 頁

請詳列計算過程，否則不予計分。

1. Find the following limits. (15%)

   (a) $\lim_{x\to 2}\dfrac{x^2-4}{x^2-3x+2}$　　(b) $\lim_{x\to 1}\dfrac{x-1}{\sqrt{x+3}-2}$　　(c) $\lim_{x\to 0}\dfrac{x\sin x}{1-\cos x}$

2. Find $\dfrac{dy}{dx}$. (20%)

   (a) $y=e^x\sqrt{x^2+1}$　　(b) $y=x^x$

   (c) $e^x-e^y=xy$　　(d) $y=\displaystyle\int_0^{x^3}\sin t^2\,dt$

3. Find the following integrals. (20%)

   (a) $\displaystyle\int \sqrt{x}+x^{-1}+\sin x\,dx$　　(b) $\displaystyle\int_{-2}^{2}\sin(x^3)+\sqrt{4-x^2}\,dx$

   (c) $\displaystyle\int \tan^3 x\,dx$　　(d) $\displaystyle\int_0^{\frac{\pi}{2}}\frac{\sin^5 x}{\sin^5 x+\cos^5 x}\,dx$

4. Determine whether the following series converge or diverge. (10%)

   (a) $\displaystyle\sum_{n=1}^{\infty}\sqrt[n]{n}$　　(b) $\displaystyle\sum_{n=1}^{\infty}\frac{n!}{n^n}$

5. Let $f(x)=x(x-4)^3$. (15%)

   (a) Find the intervals of increase or decrease.

   (b) Find the local maximum and minimum values.

   (c) Find the intervals of concavity and the inflection points.

6. Use Lagrange multipliers to find the maximum and minimum of $f(x,y)=x^2y$ subject to the constraint $x^2+2y^2=6$. (10%)

7. Evaluate the integral $\displaystyle\int_0^1\int_0^{\sqrt{1-x^2}} e^{x^2}e^{y^2}\,dydx$. (10%)