淡江大學 108 學年度日間部寒假轉學生招生考試試題

系別：數學系三年級　　科目：微積分　　24-1

考試日期：1 月 13 日(星期一) 第 2 節　　本試題共 4 大題，1 頁

＊所有計算題皆要有計算過程，否則不予計分。

1. (10 分) True or False. Explain your reason briefly.

   (a) There exists a continuous function which is not differentiable and there exists a differentiable function which is not continuous.

   (b) Denote $a < b$ two real numbers. If f is continuous in the interval (a, b) then $|f|$ is continuous in (a, b).

   (c) Denote $a < b$ two real numbers. $f : (a, b) \to \mathbb{R}$ is a differentiable function. If $f'(x_0) = 0$ for some $x_0 \in (a, b)$, then f attains local extreme value at $x = x_0$.

   (d) Suppose $|f|$ is a Riemann integrable function over $[a, b]$ then f is also Riemann integrable over $[a, b]$, where $a < b$ are two real numbers.

   (e) Denote $D \subset \mathbb{R}^2$ a open set. $f(x, y)$ is a function defined in D. Suppose $\frac{\partial f}{\partial x}(x, y)$ and $\frac{\partial f}{\partial y}(x, y)$ exist at (x_0, y_0), then $f(x, y)$ is continuous at (x_0, y_0).

2. (30 分) Find the values.

   (a) $\lim_{x\to\infty}(\sqrt{x^2+5}-x)$,

   (b) $\lim_{x\to 0}\dfrac{\tan(3x)\sin(2x)(1-\cos x)}{x^4}$,

   (c) $\lim_{n\to\infty}\sum_{i=1}^{n}\dfrac{1}{5n+i}$,

   (d) $\lim_{x\to 0^+}(\dfrac{1}{e^x-1}-\dfrac{1}{x})$,

   (e) $y(x) = x^{3x}$, $y'(2) = ?$

3. (35 分) Evaluate the integrals

   (a) $\int_0^1 \dfrac{x-4}{x^2-5x+6}\,dx$

   (b) $\int_0^{\pi/2} \cos(x)e^{2x}\,dx$.

   (c) $\int_0^{\infty} x^2e^{-x}\,dx$.

   (d) $\int_0^1\int_x^1 \sin(y^2)\,dy\,dx$.

   (e) $\iint_{\mathbb{R}^2} e^{-x^2-y^2}\,dA$.

4. (a) (8 分) Use the definition to prove $\lim_{x\to 2} x^4 = 16$.

   (b) Denote f a continuous function on $[0, 1]$. Show that

   i. (7 分) $\lim_{n\to\infty}\left(\int_0^1 x^n f(x)\,dx\right) = 0$.

   ii. (10 分) $\lim_{n\to\infty}\left(\int_0^1 |f(x)|^n\,dx\right)^{1/n} = \max_{0\le x\le 1}|f(x)|$.