

# 淡江大學 108 學年度日間部轉學生招生考試試題

系別：數學學系三年級

科目：線性代數

42-1

考試日期：1月13日(星期日) 第1節

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1. (20 %) Find the characteristic polynomial and the eigenvalues for the  $3 \times 3$

$$\text{matrix } A = \begin{bmatrix} 1 & -5 & 2 \\ 2 & 3 & 4 \\ 9 & -1 & 3 \end{bmatrix}.$$

2. (10 %) Find the inverse matrix  $B^{-1}$  for the  $3 \times 3$  matrix  $B = \begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} \end{bmatrix}$ .

3. (20 points) Let  $T$  be the linear operator on  $\mathbb{R}^3$ , the matrix of which in the standard ordered basis is

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ -1 & 3 & 4 \end{bmatrix}$$

Find a basis for the range of  $T$  and a basis for the null space of  $T$ .

4. (20 points) Let  $T$  be the linear transformation from  $\mathbb{R}^3$  into  $\mathbb{R}^2$  defined by

$$T(x_1, x_2, x_3) = (x_1 + x_2, 2x_3 - x_1).$$

If  $\mathcal{B}$  is the standard ordered basis for  $\mathbb{R}^3$  and  $\mathcal{B}'$  is the standard ordered basis for  $\mathbb{R}^2$ , what is the matrix of  $T$  relative to the pair  $\mathcal{B}, \mathcal{B}'$ ?

5. (20 points) Let  $\mathcal{B} = \{\alpha_1, \alpha_2, \alpha_3\}$  be the ordered basis for  $\mathbb{R}^3$  consisting of

$$\alpha_1 = (1, 0, -1), \quad \alpha_2 = (1, 1, 1), \quad \alpha_3 = (1, 0, 0).$$

What are the coordinates of the vector  $(a, b, c)$  in the ordered basis  $\mathcal{B}$ ?

6. (10 points) Let  $T : V \rightarrow V$  be an onto linear transformation and  $\dim(V) = n$ . Show that  $T$  is one-to-one.