

淡江大學 108 學年度日間部寒假轉學生招生考試試題

系別：物理學系三年級

科目：應用數學

9-1

考試日期：1月13日(星期一) 第2節

本試題共 4 大題，1 頁

請詳細寫出各解題步驟及計算過程，否則不予計分。

1. (25%) Given a matrix  $\begin{pmatrix} 1 & 4 & 0 \\ 4 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

- (1) (20%) find its eigenvalues and eigenvectors.
- (2) (5%) show that the corresponding eigenvectors are mutually orthogonal.

2. (25%) The differential equation is given by  $\frac{d^2 f}{dt^2} + 6\frac{df}{dt} + 9f = 4e^{-2t}$ .

- (1) (18%) Find the general solution of the equation.
- (2) (7%) When  $f=0$  and  $df/dt=2$  at  $t=0$ , find the solution of the equation.

3. (15%) Let  $f(x) = \begin{cases} 0 & -\pi < x < 0 \\ x^2 & 0 \leq x < \pi \end{cases}$ , expand  $f(x)$  as a Fourier Series.

4. (35%) Two vector fields are given by  $\mathbf{a}=(xy^2+z)\mathbf{i}+(x^2y+2)\mathbf{j}+x\mathbf{k}$  and  $\mathbf{b}=(x+y)\mathbf{i}+(y-x)\mathbf{j}$ , where  $\mathbf{i}$ ,  $\mathbf{j}$  and  $\mathbf{k}$  are unit vectors in Cartesian coordinate system.

- (1) (5%) Calculate  $\nabla \times \mathbf{a}$  and  $\nabla \times \mathbf{b}$ .
- (2) (5%) Which one (in  $\mathbf{a}$  and  $\mathbf{b}$ ) is a conservative field? Why?
- (3) (15%) Evaluate the line integral  $I = \int_A^B \mathbf{a} \cdot d\mathbf{r}$  along a straight line, where  $A=(2, 2, 1)$  and  $B=(4, 1, 1)$ .
- (4) (10%) Evaluate  $J = \int_C \mathbf{b} \cdot d\mathbf{r}$  along each of the paths in the  $xy$ -plane, namely
  - (i) the parabola  $y^2=x$  from  $(1, 1)$  to  $(4, 2)$ ,
  - (ii) the curve  $x=2u^2+u+1, y=1+u^2$  from  $(1, 1)$  to  $(4, 2)$ .