

淡江大學 108 學年度日間部寒假轉學生招生考試試題

系別：數學系資統組三年級

科目：機率與統計學

考試日期：1月13日(星期一) 第1節

本試題共 10 大題，1 頁

請詳列計算過程，否則不予計分，每題10分，共100分

1. If  $P(A)=0.4, P(B)=0.5$ , and  $P(A \cap B)=0.3$ . Find  $P(A|B)$  and  $P(B|A)$ .

2. Let  $A, B$  be independent event with probabilities  $0.7, 0.2$ , respectively. Find  $P(A \cup B)$ .

3.  $X$  has negative binomial distribution with probability mass function (pmf)

$$f(x) = \binom{x-1}{r-1} p^r (1-p)^{x-r}, \quad x=r, r+1, \dots$$
 Find the moment generating function (mgf) of  $X$ .

4. Let  $X$  and  $Y$  have the joint pmf  $f(x, y) = \frac{x+y}{21}$ ,  $x=1, 2, 3$ ,  $y=1, 2$ . Find  $E(Y|X=3)$ .

5. The probability density function (pdf) of  $X$  is  $f(x) = \frac{c}{x^2}$ ,  $1 < x < \infty$ . Calculate the value of  $c$  so that  $f(x)$  is a pdf.

6. Find  $P(X > \sqrt{Y})$  if  $X$  and  $Y$  have joint pdf  $f(x, y) = x + y$ ,  $0 \leq x \leq 1, 0 \leq y \leq 1$ .

7. Let  $X_1, \dots, X_n$  be a random sample from  $N(\theta, \sigma^2)$  with both  $\theta$  and  $\sigma^2$  unknown. Find the maximum likelihood estimation (MLE) of  $(\theta, \sigma^2)$ .

8. Let  $X$  have the uniform distribution with pdf  $f(x) = 1/4$ ,  $-1 < x < 3$ . Find the pdf of  $Y = X^2$ .

9. Let  $X_1, X_2, \dots, X_n$  be a random sample from the normal distribution  $N(\mu, 1)$ . Find the **best critical region** of size  $\alpha = 0.05$  for testing the simple hypothesis  $H_0: \mu = 10$  against the simple alternative hypothesis  $H_1: \mu = 15$ .

10. Let  $S^2$  be the variance of a random sample of size  $n$  from  $N(\mu, \sigma^2)$ . If  $n=13$  and  $12s^2 = \sum_{i=1}^{13} (x_i - \bar{x})^2 = 128.41$ . Find 90% confidence interval for  $\sigma^2$ , where  $\chi_{0.1/2}^2(12) = 21.03$  and  $\chi_{1-0.1/2}^2(12) = 5.226$ .