# 淡江大學 108 學年度日間部轉學生招生考試試題

系別：數學學系資料科學與數理統計組 三年級　　科目：機率與統計學　　23-1

考試日期：7 月 24 日(星期三) 第 1 節　　本試題共 8 大題，2 頁

本試題雙面印刷

1.(10%) Let $P(A)=0.3$ and $P(B)=0.6$ Find

(a) $P(A \cup B^c)$ if A and B are disjoint. (5%)

(b) $P(A \mid B^c)$ if A and B are independent. (5%)

2.(12%) Let $X$ be a continuous random variable with p.d.f.

$$f(x)=\begin{cases} 3/10, & -1 \le x < 0; \\ kx, & 0 \le x < 1; \\ 0, & o.w. \end{cases}$$

(a) Find k=? (5%)

(b) Find the pdf of $Y=X^2$. (7%)

3.(12%) Let $X$ be a discrete random variable with pmf $f(x)=(1/2)^x I_{\{1,2,\ldots\}}$

(a)Find $P(X>2 \mid X<4)$ (5%)

(b) Find $E(X)$ (7%)

4. (8%) Let $X_1, X_2, \ldots, X_n, X_{n+1}$ be random sample of size n+1, $n>1$，fro, a distribution that is $N(\mu,\sigma^2)$. Let $\bar{X}=\sum_{i=1}^{n} X_i/n$ and $S^2=\sum_{i=1}^{n}(X_i-\bar{X})^2/(n-1)$. Find the constant c so that the statistic $c(\bar{X}-X_{n+1})/S$ has a $t$-distribution.

5.(17%)Let $X_1, X_2$ are iid with the pdf uniform(0,$\theta$) and let R=$|X_2-X_1|$.

(a) Show that the pdf of R is

$$f(t;\theta)=\begin{cases} \dfrac{2}{\theta^2}(\theta-t) & \text{for } 0<t<\theta \\ 0 & eleswhere \end{cases} \qquad (10\%)$$

(b) Use the result in part (a) to find c so that $R<\theta<cR$ is a 81% confidence interval for $\theta$

6.(10%) Let $\bar{X}$ be the mean of a random sample of size n from a Normal $(\mu,16)$ distribution. Find the smallest sample size n such that ($\bar{X}$-1, $\bar{X}$+1) is a 0.9 level confidence interval for $\mu$. Given $P(Z<1.28)=0.90$, $P(Z<1.645)=0.95$, $P(Z<1.96)=0.975$, where Z is N(0,1) random variable.

背面尚有試題

7.(17%)An observation Z takes one of four values according to one of the three distributions shown in the following table of probabilities:

| | $\theta_0$ | $\theta_1$ | $\theta_2$ |
|---|---|---|---|
| $z_1$ | 0.2 | 0.5 | 0.3 |
| $z_2$ | 0.3 | 0.1 | 0 |
| $z_3$ | 0.1 | 0.2 | 0.4 |
| $z_4$ | 0.4 | 0.2 | 0.3 |

Consider testing $H_0:\theta=\theta_0$ against $H_1:\theta=\theta_1$ or $\theta_2$

(a)Find the likelihood ratio statistic $\Lambda$ .(5%)

(b)Determine all critical regions for Z defined by rules of the form $\Lambda<1$ (4%)

(c)Find type I and type II errors when $\theta=\theta_2$ for test in (b). (8%)

8. (14%)Let $X_1,X_2,\dots,X_n$ $n>1$ and $Y_1,Y_2,\dots,Y_m$ $m>1$ be two random sample from $N(\mu_1,\sigma_1^2)$ and $N(\mu_2,\sigma_2^2)$, respectively. Let $\bar{X}=\sum_{i=1}^{n}X_i/n$, $S_{XX}=\sum_{i=1}^{n}(X_i-\bar{X})^2$, $\bar{Y}=\sum_{i=1}^{m}Y_i/m$ and $S_{YY}=\sum_{i=1}^{m}(Y_i-\bar{Y})^2$

(a) Suppose n=26, m=18, $S_{XX}$=900 and $S_{YY}$=170. Use these data to test $H_0:\sigma_1=\sigma_2$ v.s. $H_1:\sigma_1\neq\sigma_2$ (with significance level 0.05). Given.

$F_{0.025,26,18}=0.43$, $F_{0.05,26,18}=0.50$, $F_{0.95,26,18}=2.13$, $F_{0.975,26,18}=2.48$

$F_{0.025,25,17}=0.42$, $F_{0.05,25,17}=0.49$, $F_{0.95,25,17}=2.18$, $F_{0.975,25,17}=2.55$

(b) Suppose $\sigma_1=\sigma_2$. n=10, m=8, $\bar{X}=120$, $\bar{Y}=116$, $S_{XX}$=56 and $S_{YY}$=40 . Use these data to test $H_0:\mu_1=\mu_2$ v.s. $H_1:\mu_1>\mu_2$ (with significance level 0.05). Given.

$T_{0.025,18}=-2.10$, $T_{0.05,18}=-1.73$,

$T_{0.025,17}=-2.11$, $T_{0.05,17}=-1.74$,

$T_{0.025,16}=-2.12$, $T_{0.05,16}=-1.75$,