

淡江大學 108 學年度日間部轉學生招生考試試題

系別：數學學系數學組三年級

科目：線性代數

22
1 頁

考試日期：7 月 24 日(星期三) 第 1 節

本試題共 7 大題，

Partial credit—You must show all your work.

1. (20 %) Find the characteristic polynomial and the eigenvalues for the 3×3 matrix $A = \begin{bmatrix} 0 & -2 & -3 \\ -1 & 1 & -1 \\ 2 & 2 & 5 \end{bmatrix}$.

2. (10 %) Find the inverse matrix B^{-1} for the 3×3 matrix $B = \begin{bmatrix} -3 & -5 & -7 \\ 2 & 4 & 3 \\ 0 & 1 & -1 \end{bmatrix}$.

3. (20 %) Let $A = \begin{bmatrix} -1 & 1 & 0 \\ 0 & 3 & 0 \\ 4 & -2 & 5 \end{bmatrix}$. Find an invertible 3×3 matrix P and a 3×3 diagonal matrix D such that $P^{-1}AP = D$.

4. (10 %) Are the vectors

$$\alpha_1 = (2, 4, 1, 1), \quad \alpha_2 = (-5, 2, 2, -1),$$

$$\alpha_3 = (-4, 0, 1, -1), \quad \alpha_4 = (1, 6, 2, 1)$$

linearly independent in \mathbb{R}^4 ? Write down your argument.

5. (10 %) Let W_1 be the set of matrices of the form

$$\begin{bmatrix} x & -x \\ y & z \end{bmatrix}, \quad x, y, z \in \mathbb{R}$$

and let W_2 be the set of matrices of the form

$$\begin{bmatrix} a & b \\ -a & c \end{bmatrix}, \quad a, b, c \in \mathbb{R}.$$

Find the dimensions of W_1 , W_2 , and $W_1 \cap W_2$.

6. Let T be the linear operator on \mathbb{R}^3 defined by

$$T(x, y, z) = (3x + z, -2x + y, -x + 2y + 4z).$$

(a) (10 %) What is the matrix of T in the standard ordered basis $\{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$ for \mathbb{R}^3 ?

(b) (10 %) What is the matrix of T in the ordered basis $\{\alpha_1, \alpha_2, \alpha_3\}$ where $\alpha_1 = (1, 0, 1)$, $\alpha_2 = (0, 1, 1)$, and $\alpha_3 = (0, 0, 1)$?

7. (10 %) Let $T : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ be a one-to-one linear transformation. Show that T is onto.