

# 淡江大學 108 學年度碩士班招生考試試題

系別：數學學系

科目：微積分

4-1

考試日期：3 月 10 日(星期日) 第 1 節

本試題共 8 大題， 1 頁

請詳列演算過程，否則不予計分。

1. Evaluate the following limits.

a)  $\lim_{x \rightarrow -3} \frac{x+3}{2x^2+5x-3}$  (6%)      b)  $\lim_{x \rightarrow -\infty} \frac{\sqrt{5x^6-x}}{3x^3+1}$  (6%)

2. Find the equation of the line tangent to the graph of  $2x+3y = \cos(xy^2)$  at the point  $(\frac{\pi}{2}, 1)$ .  
(10%)

3. Evaluate the following integrals

a)  $\int_1^3 (2x+1)(x-1)^{10} dx$  (7%)      b)  $\int (\ln x)^2 dx$  (7%)

4. Determine whether the following series converges or diverges?

a)  $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{1+n^2}$  (7%)      b)  $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$  (7%)

5. a) Find the Maclaurin series of  $f(x) = \frac{\sin(2x)}{x}$  (5%)

b) Use the power series obtained in a) to approximate the value of  $\int_0^{0.5} \frac{\sin 2x}{x} dx$  accurate to third decimal place. (8%)

c) Find  $f^{(99)}(0)$ . (5%)

6. Show that if  $w(r, s, t) = f(2r-2s, s-t, 3t-3r)$  then  $\frac{\partial w}{\partial r} + \frac{\partial w}{\partial s} + \frac{\partial w}{\partial t} = 0$  (8%)

7 Evaluate the following double integral

a)  $\iint_D x^2 y \sqrt{1+x^3} dA$  where  $D = \{(x, y); 0 \leq x \leq 2, 0 \leq y \leq 1\}$  (7%)

b)  $\int_0^1 \int_y^1 \frac{1}{1+x^2} dx dy$ . (7%)

8 Find the volume of the solid bounded above by the sphere  $x^2 + y^2 + z^2 = 8$ , below by the plane  $z = 4$ . (10%)