淡江大學 108 學年度碩士班招生考試試題

系別:航空太空工程學系 科目:工程數學 13-1 考試日期:3月10日(星期日)第1節 本試題共 3 大題, 1 頁

1. Consider the following ordinary differential equation of initial value problem,

$$y'' + 5y' + 6y = 3x \tag{1}$$

where y is differentiated with respect to x. Let y(0) = 1 and y'(0) = 1, please answer the following questions:

- (a) (20 %) Please solve this problem by assuming $y_h(x) = e^{\lambda x}$ along with the method of undetermined coefficients, where $y_h(x)$ denotes a homogeneous solution.
- (b) (20 %) Please solve this problem by Laplace Transform.
- 2. Consider the following ordinary differential equation of initial value problem,

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} \tag{2}$$

where $\mathbf{x} = \begin{bmatrix} x_1 & x_2 \end{bmatrix}^T$ and

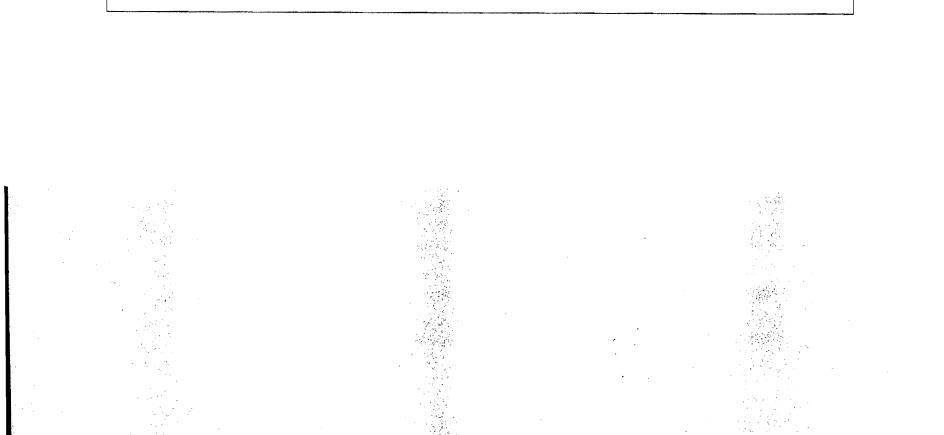
$$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ -8 & -6 \end{bmatrix}$$

with initial conditions $x_1(0) = -1$ and $x_2(0) = 1$.

- (a) (20 %) Please find the eigenvalues and the corresponding eigenvectors of matrix
 A. Remember to represent the eigenvectors as unit vectors.
- (b) (20 %) Please solve this ODE problem.
- 3. (20 %) A one-dimensional heat problem is described by a partial differential equation (PDE), given by

$$\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2} \tag{3}$$

where u(x,t) denotes the temperature in the location x at time t, and c is a constant. Given the boundary conditions: u(0,t) = 0, u(L,t) = 0 for all t, and the initial condition $u(x,0) = \sin(x\pi/L)$, please solve the PDE for u(x,t).



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