1. Find the general solution of the following differential equation. (15%)

$$y''+5y'+6y = 0$$

2. Find the solution of the initial value problem in the following differential equation. (20%)

$$y''+3y'+2y = 2, \quad y(0) = 2, \ y'(0) = 0$$

3. Consider the following system of linear equations $(Ax = b)$.

$$
\begin{array}{rcrcrcr}
 & & x_2 & + & 2x_3 & = & 8 \\
-2x_1 & + & 4x_2 & - & 2x_3 & = & 0 \\
2x_1 & - & 5x_2 & + & 4x_3 & = & 4
\end{array}
$$

   (a) Find the coefficient matrix A of this system. (5%)

   (b) Find the augmented matrix of this system. (5%)

   (c) Use Gauss-Jordan Elimination to find the solution of this system. (15%)

4. Consider the matrix $A = \begin{bmatrix} -1 & -2 \\ 3 & 4 \end{bmatrix}$.

   (a) Find the characteristic equation of A and the eigenvalues of A, where $\lambda_1 \leq \lambda_2$. (10%)

   (b) Find the eigenvectors of A corresponding to each eigenvalue of A. (10%)

   (c) Find an invertible matrix P and its inverse P^{-1} such that $P^{-1}AP = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$. (10%)

   (d) Find A^{10}. (10%)