# 淡江大學 108 學年度碩士班招生考試試題

系別：數學學系　　　　科目：統計學　　　　24-1

考試日期：3 月 10 日(星期日) 第 2 節　　　　本試題共 6 大題， 1 頁

**1.** (10%) If $X$ has distribution given by $P(X=0)=P(X=2)=p$ and $P(X=1)=1-2p$ for $0 \le p \le 1/2$, for what $p$ is the variance of $X$ a maximum?

**2.** (20%) **a)** State the central limit theorem, and Chebyshev's theorem. **b)** A random sample of size $n=81$ is taken from an infinite population with the mean 128 and standard deviation 6.3. With what probability can we assert that the value we obtain for $\overline{X}$ will not fall between 126.6 and 129.4 if we use the central limit theorem; and Chebyshev's theorem? [You may use the c.d.f. $\phi(z)$ of $N(0,1)$ for your answer.]

**3.** (10%) Consider two random variables $X$ and $Y$ with the joint p.d.f. $f(x,y)=12xy(1-y)$, $0<x<1, 0<y<1$. Find the probability density of $Z=XY^2$.

**4.** (30%) Let $X_1,\cdots,X_n$ be a random sample from distribution with the p.d.f. $f(x)=\frac{1}{\theta}e^{-x/\theta}$, $0<x<\infty, \theta>0$. **a)** Find the moment-generating function, and what are the mean and variance? **b)** Find the maximum likelihood estimate of $\theta$. Is it a sufficient estimator of $\theta$? Why?

**5.** (20%) Let $X_1, X_2$ denote a random sample of size 2 from the distribution in Problem **4**. Consider the simple hypothesis $H_0:\theta=2$ against $H_1:\theta=4$. **a)** Show that the best test of $H_0$ against $H_1$ may be the statistic $X_1+X_2$. **b)** Consider a critical region $C=\{(x_1,x_2): x_1+x_2 \ge 9\}$, find the significant level and the power function of this test.

**6.** (10%) Let $X_1,\cdots,X_n$ be a random sample from distribution with the p.d.f. $f(x)=1/\theta$, $0<x<\theta$. Show that the largest sample value, i.e., the $n$th order statistic $Y_n$, is a biased estimator of $\theta$. And modify this estimator of $\theta$ to make it unbiased.