淡江大學 107 學年度日間部寒假轉學生招生考試試題

系別： 物理系三年級　　科目：應用數學　　27-1

考試日期：1 月 13 日(星期日) 第 2 節　　本試題共 4 大題， 1 頁

1 (10%+10%) Consider an ordinary differential equation (ODE) given by $\boxed{(1-xy)\frac{dy}{dx}+y^2+3xy^3=0}$.

(a) Find the integration factor $\mu(x,y)$ to turn this ODE into an exact differential equation. [Hint: Try $\mu(x,y)=x^{\alpha}y^{\beta}$ and determine α and β.]

(b) Solve this differential equation.

2 (10%+10%+10%)

(a) Obtain the general solution of the homogeneous ODE $\boxed{y''-4y'+4y=0}$.

(b) Find a particular solution of the inhomogeneous ODE $\boxed{y''-4y'+4y=xe^{2x}}$.

(c) Find the solution of $\boxed{y''-4y'+4y=xe^{2x}}$ satisfying the initial conditions: $y(x=0)=0$ and $y'(x=0)=1$.

3 (20%) Determine the series solution of the ODE $\boxed{y''-xy=0}$ satisfying the initial conditions: $y(x=0)=1$ and $y'(x=0)=0$.

4 (12%+6%+12%)

(a) Find the sine/cosine Fourier series for the function $f(x)$ with the period of 2 defined as $x+1$ for $-1\le x<0$ and $-x+1$ for $0\le x<1$.

(b) Using the Fourier expansion obtained for $f(x)$ in (a), find the sine/cosine Fourier series for the function $f(x)$ with the period of 2 defined as 0 for $-1\le x<0$ and 1 for $0\le x<1$.

(c) Use the Fourier series in (a) and (b) to show that

$$S_1=1+\frac{1}{3^2}+\frac{1}{5^2}+\frac{1}{7^2}+\cdots=\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}=\frac{\pi^2}{8}\quad\text{and}$$

$$S_2=1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots=\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{2n-1}=\frac{\pi}{4}.$$