

淡江大學 107 學年度日間部寒假轉學生招生考試試題

系別：數學系三年級

科目：微積分

26-

考試日期：1月13日(星期日) 第2節

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#注意：務必要有計算過程，否則不予計分

1. Let $F(x) = g(f(x))$, where f and g are differentiable on $(-\infty, \infty)$. If $f(2)=3$, $f'(2)=-3$, and $g'(3)=4$, find $F'(2)$. (8points)

2. Let $f(x) = \int_0^x \sqrt{9+t^2} dt$. Find $f'(4)$. (8points)

3. Find an equation of the tangent line to the curve $y = 3x^2 + 2x - 1$ at the point $(1, 4)$. (8points)

4. Find the following limits (16points) :

(1) $\lim_{x \rightarrow 1} \frac{\sqrt{x}-1}{x-1}$.

(2) $\lim_{x \rightarrow 1} \left(\frac{\ln x}{x-1} \right)$.

5. Find the following integrals (32points)

(a) $\int_0^8 |\sqrt[3]{x}-1| dx$.

(b) $\int \left(\frac{\sqrt{x}}{2} + \frac{2}{\sqrt{x}} \right) dx$.

(c) $\int x \cos x dx$.

(d) $\int (\sin x)^3 (\cos x)^2 dx$.

6. Let $f(x) = x + \frac{4}{x}$. Find the absolute maximum and absolute minimum values of f on $[1, 5]$. (8points)

7. Evaluate $\iint_R x e^{xy} dA$, where R is the rectangular $0 \leq x \leq 2$ and $0 \leq y \leq 1$. (8points)

8. Find $\frac{dy}{dx}$ if (12points) :

(a) $y = \sqrt{\frac{x-1}{x+1}}$.

(b) $y = x^{\sqrt{x}}$.