10

淡江大學 107 學年度日間部寒假轉學生招生考試試題

系別：數學系資統組三年級　　科目：機率與統計學　10-1

考試日期：1 月 13 日(星期日) 第 1 節　　本試題共 6 大題， 2 頁

本試題雙面印刷

1. Let A, B, C be three events, $P(A) = 0.5, P(B|A) = 0.2, P(A^c|B^c) = 0.2$,
   (1) find $P(A \cap B) =?$ (5%)
   (2) find $P(B) =?$ (5%)

2. Let $f(x, y) = c(x + 2y)$, $(x, y) = (1,1), (1,2), (2,1), (2,2)$ be the joint pmf of two random variables X and Y of the discrete type. Find
   (1) $c =?$ (5%)
   (2) covariance of X and Y. (5%)
   (3) $\mathrm{E}(Y|X = 2)$ (5%)

3. Let $f(x; \theta) = (1/\theta)x^{(1-\theta)/\theta}$, $0 < x < 1$, $0 < \theta < \infty$. Let $X_1, X_2, \ldots, X_n$ denote a random sample of size n from this distribution.
   (1) Find the maximum likelihood estimator of θ. (5%)
   (2) Let $Y = -\ln X_1$, find the pdf of Y. (5%)
   (3) Find the variance of the maximum likelihood estimator of θ. (10%)

3.Let $X_1, X_2, \ldots, X_n$ be random sample from uniform distribution on the interval $(\theta - 1, \theta + 1)$.
   (1) Find $\mathrm{E}(X_1)$ and $\mathrm{E}(X_1^2)$ (5%)
   (2) Find the method of moments estimator for θ (5%)

4. Let X equal the tarsus length for a male grackle. Assume that the distribution of X is $N(\mu, 2.2^2)$. Find the sample size n that is needed so that we are 95% confident that the maximum error of the estimate of μ is 0.4. (10%)

5. Assume that SAT mathematics scores of students who attend small liberal arts colleges are $N(\mu, 90^2)$. We shall test $H_0: \mu = 530$ against the alternative hypothesis $H_1: \mu < 530$. Given a random sample of size n=36 SAT mathematics scores, let the critical region be defined by $C = \{\bar{x} | \bar{x} \leq 510.77\}$, where $\bar{x}$ is the observed mean of the sample.
   (a) How is the power function, $\mathrm{K}(\mu)$, defined for this test? (5%)
   (b)What is the significance level of this test? (5%)
   (c)Sketch the graph of the power function. (5%)
   (d)What is the p_value corresponding to $\bar{x} = 83.41$(5%)

6. Let X,Y denote the tarsus lengths of male and female grackles, respectively. Assume that X is $N(\mu_x,\sigma_x^2)$ and Y is $N(\mu_y,\sigma_y^2)$. Given that n=25, $\bar{x}=33.8$, $s_x^2=4.88$, m=29, $\bar{y}=31.66$, and $s_y^2=5.81$, test

(a) $H_0: \frac{\sigma_x^2}{\sigma_y^2}=1$ against a two side alternative with $\alpha=0.02$. (7%)

(b) $H_0: \mu_x=\mu_y$ against $H_1: \mu_x>\mu_y$ with $\alpha=0.01$. (8%)

Note: Suppose $Z\sim N(0,1)$ and F_{r_1,r_2}~F distribution with r_1 *and* r_2 degrees of freedom. then P(|Z|≤ 1.96)=0.95, P(|Z|≤ 1.645)=0.90, P(Z ≤ 2.326)=0.99

P($F_{28,24}\le 0.4$)=0.01, and P($F_{28,24}\ge 2.6$)=0.01