Please show all your work to receive full credit.

1. (10 pts) Write down a basis for the space of all 3×3 real symmetric matrices.

2. (15 pts) Let $C(-\infty, \infty)$ be the space of all continuous functions defined on $\mathbb{R}$. Determine if the following sets of vectors in $C(-\infty, \infty)$ are linearly independent.
(a) $\{(x-2)^2, x^2-4x, 3\}$　　　(b) $\{1, x, e^x\}$

3. (15 pts) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation defined by $T(x,y) = (5x+2y, -x+2y)$. Find a basis for $\mathbb{R}^2$ relative to which the matrix representation for T is diagonal.

4. (15 pts) Let $A = \begin{bmatrix} 1 & 2 & 3 & 5 \\ 2 & 1 & 1 & 2 \\ -5 & 2 & 5 & 7 \end{bmatrix}$ and let $\mathbf{b}$ be a 3×1 matrix.
(a) If $A\mathbf{x} = \mathbf{b}$ has a solution, determine the rank of the matrix $[A \,|\, \mathbf{b}]$.
(b) If $A\mathbf{x} = \mathbf{b}$ has no solution, determine the rank of the matrix $[A \,|\, \mathbf{b}]$.
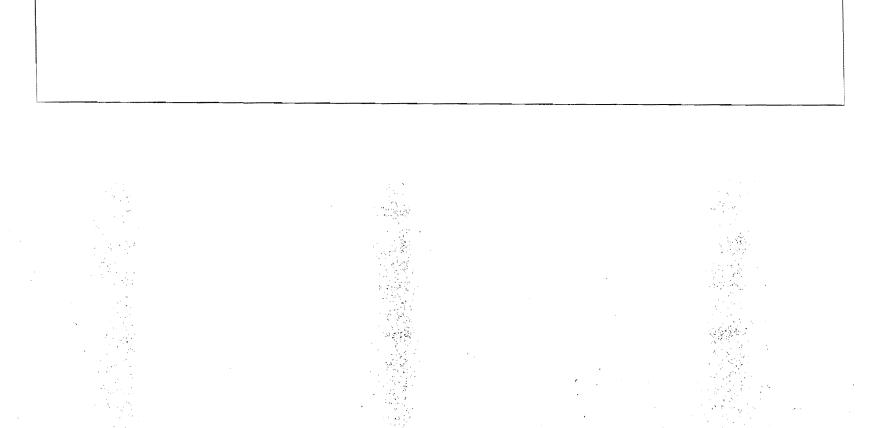Please justify your answer.

5. (15 pts) Let W be the one dimensional subspace spanned by the vector $(1, 2, 3, 0)$ in $\mathbb{R}^4$. Find a basis for $W^\perp$ (the orthogonal complement of W).

6. (15 pts) Suppose $M_{2\times2}$ is the space of all 2×2 matrices with real entries. Let $T : M_{2\times2} \to M_{2\times2}$ be the linear transformation given by $T(A) = A^T$ (where A^T is the transpose of A). Find the characteristic polynomial of T.

7. (15 pts) Suppose A and B are both 3×3 matrices with real entries.
(a) Show that if A and B are similar matrices, then A^n and B^n are also similar matrices for any $n \in \mathbb{N}$.
(b) Give an example showing that if A^2 and B^2 are similar, A and B need not be similar.