1. Solve the 2nd order ordinary differential equation $y'' + x^2 y = 0$ by the series solution method. (25%)

2. Solve the 2nd order ordinary differential equation $y'' + 4y = x + 2e^{-2x}$. (25%)

3. Find the eigenvalues and eigenvectors of the matrix (25%)

$$\mathbf{A} = \begin{pmatrix} -5 & 2 \\ 2 & -2 \end{pmatrix}$$

4. (1) Find the Laplace transform of the function $(t^3 - 3t + 2)e^{-2t}$. (12%)

   (2) Find the inverse Laplace transform of $\dfrac{4}{s^2+4s+20}$. (13%)

Table of Laplace Transforms

$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$	$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$
1. 1	$\dfrac{1}{s}$	2. e^{at}	$\dfrac{1}{s-a}$
3. $t^n,\ n=1,2,3,\ldots$	$\dfrac{n!}{s^{n+1}}$	4. $t^p, p > -1$	$\dfrac{\Gamma(p+1)}{s^{p+1}}$
5. $\sqrt{t}$	$\dfrac{\sqrt{\pi}}{2s^{\frac{3}{2}}}$	6. $t^{n-\frac{1}{2}},\ n=1,2,3,\ldots$	$\dfrac{1\cdot 3\cdot 5\cdots(2n-1)\sqrt{\pi}}{2^n s^{n+\frac{1}{2}}}$
7. $\sin(at)$	$\dfrac{a}{s^2+a^2}$	8. $\cos(at)$	$\dfrac{s}{s^2+a^2}$
9. $t\sin(at)$	$\dfrac{2as}{(s^2+a^2)^2}$	10. $t\cos(at)$	$\dfrac{s^2-a^2}{(s^2+a^2)^2}$
11. $\sin(at)-at\cos(at)$	$\dfrac{2a^3}{(s^2+a^2)^2}$	12. $\sin(at)+at\cos(at)$	$\dfrac{2as^2}{(s^2+a^2)^2}$
13. $\cos(at)-at\sin(at)$	$\dfrac{s(s^2-a^2)}{(s^2+a^2)^2}$	14. $\cos(at)+at\sin(at)$	$\dfrac{s(s^2+3a^2)}{(s^2+a^2)^2}$
15. $\sin(at+b)$	$\dfrac{s\sin(b)+a\cos(b)}{s^2+a^2}$	16. $\cos(at+b)$	$\dfrac{s\cos(b)-a\sin(b)}{s^2+a^2}$
17. $\sinh(at)$	$\dfrac{a}{s^2-a^2}$	18. $\cosh(at)$	$\dfrac{s}{s^2-a^2}$
19. $e^{at}\sin(bt)$	$\dfrac{b}{(s-a)^2+b^2}$	20. $e^{at}\cos(bt)$	$\dfrac{s-a}{(s-a)^2+b^2}$