

# 淡江大學 107 學年度日間部轉學生招生考試試題

系別：物理學系三年級

科目：應用數學

42-1

考試日期：7月27日(星期五) 第2節

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1. Find the inverse matrix of (8%)

$$\begin{pmatrix} 3 & 2 & 1 \\ 2 & 2 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

2. The three Pauli matrices are

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

Find the eigenvalues and eigenvectors of  $\sigma_x$ ,  $\sigma_y$  and  $\sigma_z$  respectively. (12%)

3. Given the three vectors

$$\begin{aligned} \vec{P} &= 3\hat{x} + 2\hat{y} - \hat{z} \\ \vec{Q} &= -6\hat{x} - 4\hat{y} + 2\hat{z} \\ \vec{R} &= \hat{x} - 2\hat{y} - \hat{z} \end{aligned}$$

find two that are perpendicular (10%) and two that are parallel or antiparallel (10%)

4. From Kirchhoff's law the current  $I$  in an RC (resistance-capacitance) circuit obeys the equation

$$R \frac{dI}{dt} + \frac{1}{C} I = 0$$

(a) Find  $I(t)$  (12%)

(b) For a capacitance of 10,000 microfarads charged to 100 volts and discharging through a resistance of 1 megohm, find the current  $I$  for  $t=0$  and for  $t=100$  seconds. (8%)

5. The force field acting on a two-dimensional linear oscillator may be described by

$$\vec{F} = -\hat{x}kx - \hat{y}ky$$

Compare the work done moving against this force when going from (1,1) to (4,4) by the following straight-line path :

(a) (1,1)  $\rightarrow$  (4,1)  $\rightarrow$  (4,4) (10%)

(b) (1,1)  $\rightarrow$  (4,4) along  $x=y$  (10%)

This means evaluating

$$-\int_{(1,1)}^{(4,4)} \vec{F} \cdot d\vec{r}$$

along each path.

6. A symmetric triangular pulse of adjustable height and width is described by

$$f(x) = a(1 - |x|/b) \text{ for } 0 \leq |x| \leq b \text{ and } f(x) = 0 \text{ for } b \leq |x| \leq \pi$$

Show that the Fourier coefficients are (20%)

$$a_0 = \frac{ab}{\pi} \quad \text{and} \quad a_n = \frac{2ab}{\pi} (1 - \cos nb) / (nb)^2$$