

淡江大學 107 學年度日間部轉學生招生考試試題

系別：數學學系資料科學與数理統計組
三年級

科目：機率與統計學 23-1

考試日期：7月27日(星期五) 第1節

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- (10%) Let $P(A) = 0.3$ & $P(B) = 0.6$. Find (a) $P(A \cup B)$ if A & B are independent. (b) $P(A^c | B)$ if A & B are disjoint.
- (15%) Suppose X and Y are independent with $EX = 2, EY = 1, V(X) = 1, V(Y) = 3$. Find (a) $E(3X + 2Y)$ (b) $V(3X - 2Y)$ (c) $V(XY)$ (d) $EE(Y | X)$ (e) $\rho(X, Y)$
- (10%) The moment generating function of a random variable X is $M(t) = \frac{2}{5}e^t + \frac{1}{5}e^{2t} + \frac{2}{5}e^{3t}$, $t \in \mathbb{R}$. Find the probability mass function of X , EX , and $V(X)$.
- (15%) (a) State the Central Limit Theorem (CLT). (b) Given a pdf $f(x) = x^3 / 4, 0 < x < 2$. Find the mean and variance of this distribution. (c) Let \bar{X} be the mean of a random sample of size 24 from f in (b), use CLT to approximate $P(\bar{X} > 2)$. [use the cdf $\phi(z)$ of $N(0, 1)$ to answer (c)]
- (10%) Let X_1, \dots, X_n be a random sample from the pdf $f(x) = \theta x^{\theta-1}, 0 < x < 1, \theta > 0$. (a) Find the moment estimator of θ . (b) Is the estimator in (a) unbiased for θ ? Why?
- (10%) Let X_1, \dots, X_{25} be a random sample from $N(\mu, \sigma^2)$ with $\bar{x} = 18, s^2 = 9$. Derive and find a 90% confidence interval for μ , where $t_{0.05}(24) = 1.7$.
- (18%) Let X_1, \dots, X_9 be a random sample from $N(\theta, 81)$. For $H_0: \theta = 75$ v.s. $H_1: \theta = 70$. (a) Given a test: reject H_0 if $\bar{X} \leq 72$. Find the probabilities of type I and type II error of this test. [use the cdf $\phi(z)$ of $N(0, 1)$ to answer them] (b) Derive the level 0.05 test. ($z_{0.025} = 1.96$).
- (12%) Given the grades (A-D levels) of 20 students from each groups. Use the contingency table to test H_0 : the grade distributions of two groups are the same (with significance level 0.05). (assume $\chi_{0.05}^2(3) = 7.8, \chi_{0.05}^2(4) = 9.5, \chi_{0.05}^2(6) = 12.6, \chi_{0.05}^2(7) = 14.1, \chi_{0.05}^2(8) = 15.5$)

| | A | B | C | D | totals |
|----------|---|---|---|----|--------|
| Group I | 2 | 4 | 4 | 10 | 20 |
| Group II | 8 | 6 | 6 | 0 | 20 |