

# 淡江大學 107 學年度日間部轉學生招生考試試題

系別：數學學系三年級

科目：微積分

41-1

考試日期：7月27日(星期五) 第2節

本試題共 9 大題， 1 頁

請務必附上計算過程，否則不予計分。

1. (18 pts) Find the limit if it exists, or show that the limit does not exist.

(a)  $\lim_{x \rightarrow \infty} (\sqrt{4x^2 + 3x + 1} - 2x)$       (b)  $\lim_{(x,y) \rightarrow (0,0)} \frac{2xy^3}{x^2+y^6}$

(c)  $\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{n}{n^2 + i^2}$

2. (12 pts) Evaluate the following integrals.

(a)  $\int \frac{2x}{x^2-x-6} dx$       (b)  $\int_0^1 \int_y^1 \frac{\sin x}{x} dx dy$

3. (10 pts) Calculate the indicated derivatives.

(a) Let  $f(x, y) = x^3 + y^3 - 2xy$ . Find  $\frac{\partial f}{\partial x}$ .

(b) Let  $g(x) = \int_{2x}^{3x} e^{t^2} dt$ . Find  $g'(x)$ .

4. (10 pts) Determine if the given series is convergent or divergent. Please provide a reason.

(a)  $\sum_{n=1}^{\infty} (-1)^n \frac{n}{\sqrt{n^2+1}}$       (b)  $\sum_{n=1}^{\infty} \frac{n5^n}{3^{2n}}$

5. (10 pts) Find an equation of the tangent plane to the surface  $xy + yz + xz = 3$  at the point  $(1, 1, 1)$ .

6. (10 pts) Find the absolute maximum and absolute minimum values of  $f(x, y) = 2x^3 + y^4$  on the disk  $x^2 + y^2 \leq 1$ .

7. (10 pts) Suppose  $f(x)$  is a differentiable function. If  $f(0) = 1$  and  $3 \leq f'(x) \leq 5$  for all values of  $x$ , show that  $4 \leq f(1) \leq 6$ .

8. (10 pts) Find the Maclaurin series for the function  $f(x) = x^2 \sin 2x$ .

9. (10 pts) Sketch the polar curve  $r = 1 + \cos \theta$  and find the area of the region enclosed by the curve.