

淡江大學 107 學年度日間部轉學生招生考試試題

系別： 數學學系數學組三年級

科目：線性代數

22-1

考試日期：7月27日(星期五) 第1節

本試題共 7 大題，2 頁

本試題雙面印刷

#務必書寫過計算程，否則不予計分。

1. Let $A = \begin{bmatrix} 6 & -5 \\ 2 & -1 \end{bmatrix}$.

- (1) Find characteristic polynomial of A. (5 Points)
- (2) Find all eigenvalues and eigenvectors of A. (5Points)
- (3) Find an invertible matrix P such that $P^{-1}AP = D$ is a diagonal matrix.
(5Points)
- (4) Find A^{10} . (5 points)

2. Let $P = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & -1 \end{bmatrix}$.

Show that P is invertible and find P^{-1} . (15 points)

3. Let M be the vector space of all 2×2 matrices. Let $A = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ and let

$$U = \{X \text{ in } M \mid AX = XA\}.$$

- (1) Show that U is a subspace of M. (5points)
- (2) Find a basis for U. (5points)

4. Let $a_1 = (0, 1, 1), a_2 = (1, 1, 0), a_3 = (0, 1, 0)$. Show that $S = \{a_1, a_2, a_3\}$ form a basis for \mathbb{R}^3 .

(10 points)

背面尚有試題

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5. Let $u_1 = (1, 1)$ and $u_2 = (1, -1)$, and let $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be the linear operator such that

$$T(u_1) = (1, -2) \text{ and } T(u_2) = (-4, 1)$$

Find a formula for $T(x, y)$. (15 points)

6. Let $A = \begin{bmatrix} 1 & -1 & 0 & 2 \\ 0 & -2 & 2 & 4 \\ 1 & -1 & 0 & 3 \end{bmatrix}$ be 3×4 matrix. (20 points)

(a) Show that $AX=Y$ is consistent for all 3×1 matrix Y .

(b) Find a basis for the solution space of $AX=0$.

7. Let A be $m \times n$ and B be $n \times m$ matrices. Prove that if $m < n$, then BA is not invertible.

(10 points)