淡江大學 107 學年度碩士班招生考試試題

27-1

系別：數學學系　　　　科目：統計學

考試日期：3 月 11 日(星期日) 第 2 節　　　　本試題共 6 大題， 1 頁

1. (10%) Let $X_{(1)} < X_{(2)} < \ldots < X_{(n)}$ be the order statistics of a random sample of size n from an exponential distribution with pdf $f(x) = e^{-x}, 0 < x < \infty$. What is the distribution of $U = e^{-X_{(r)}}$.

2. (20%) Let $X_1, X_2, \ldots, X_n$ be a random sample from a gamma distribution with known parameter α and unknown parameter $\theta > 0$.

a) (10%) Show that $Y = \sum_{i=1}^n X_i$ is a complete and minimal sufficient statistic for θ.

b) (10%) Let $Z = \dfrac{\sum_{i=1}^n a_i X_i}{\sum_{i=1}^n X_i}$, where $a_1, a_2, \ldots, a_n$ are not all equal constants. Are Z and Y in **(b)** independent? Why?

3. (30%) Consider the regression model $Y_i = \beta x_i + \epsilon_i$, $i = 1, \ldots, n$, where ϵ_i are i.i.d. $N(0, \sigma^2)$, σ^2 unknown.

a) (10%) Obtain the maximum likelihood estimators, $\hat{\beta}$ and $\hat{\sigma}^2$, of β and σ^2 under this model.

b) (10%) Find the distribution of $\hat{\beta}$.

c) (10%) Find the distribution of $\hat{\sigma}^2$.

4. (10%) Let $X_1, \ldots, X_n$ be i.i.d. Bernoulli(θ) where $0 \le \theta \le 1/2$. Find the maximum likelihood estimator of θ.

5. (20%) Assume that the weight of cereal in a "12.6-ounce box" is $N(\mu, 0.04)$. The Food and Drug Association allows only a small percentage of boxes to contain less than 12.6 ounces. We shall test the null hypothesis $H_0 : \mu = 13$ against the alternative hypothesis $H_1 : \mu < 13$.

a) (5%) Use a random sample of $n = 100$ to define the test statistic and the critical region that has a significance level of $\alpha = 0.05$.

b) (5%) If $\bar{x} = 12.9608$, what is your conclusion?

c) (5%) What is the p-value of this test?

d) (5%) Is this test uniformly most powerful? Why?

6. (10%) Let X be a single observation from the beta($\theta, 1$) pdf. Let $Y = -(\ln X)^{-1}$. Evaluate the confidence coefficient of the set $[y/2, y]$.