務必書寫計算過程於答案卷上，否則不予計分。

1. (a) Find $\lim\limits_{x \to 1} \left(\dfrac{1}{\ln x} - \dfrac{1}{x-1} \right)$. (6points)　(b) Find $\lim\limits_{x \to \infty} \left(\sqrt{x^2+1} - x \right)$. (6points)

2. Evaluate the following integrals

   (a) $\displaystyle\int_0^3 \dfrac{x}{\sqrt{16+x^2}}\, dx$. (6points) (b) $\displaystyle\int \sqrt{x}\, \ln x\, dx$. (6points)

   (c) $\displaystyle\iint_E y\, dA$, where E is the region that lies in the upper half – plane bounded by the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$. (6points)

   (d) $\displaystyle\int_0^4 \int_{\frac{x}{2}}^2 e^{y^2}\, dy\, dx$. (6points)

3. Determine whether the following series or improper integral is convergent or divergent.(判斷下列瑕積分或級數收斂或發散)

   (a) $\displaystyle\int_1^\infty \dfrac{1}{1+x^3}\, dx$. (6points) (b) $\displaystyle\sum_{n=1}^\infty \dfrac{3n-2}{n^3-2n^2+11}$. (6points) (c) $\displaystyle\sum_{n=1}^\infty \dfrac{n!}{(2n+1)!}$. (6points)

4. Find $\dfrac{dy}{dx}$ at x=1 if

   (a) $y = \displaystyle\int_2^{1/x} \sin^4 t\, dt$. (8points) (b) $y = (x^{3/4}\sqrt{x^2+1})/(3x+2)^2$. (8points)

5. Prove that if f is differentiable at c, then f is continuous at c. (8points)

6. Find $\dfrac{dy}{dx}$ if $x^3 + y^3 = 2xy$ and find the tangent to the curve of $x^3 + y^3 = 2xy$ at the point (1, 1). (12points)

7. If f is differentiable on $(-\infty, \infty)$ and z = f(x-y), show that $\dfrac{\partial z}{\partial x} + \dfrac{\partial z}{\partial y} = 0$. (10points)