

淡江大學 106 學年度日間部寒假轉學生招生考試試題

19-1

系別：數學學系數學組二年級

科目：微 積 分

考試日期：1月6日(星期六) 第2節

本試題共 10 大題， 1 頁

計算題(每題 10 分，共 100 分)(寫出計算過程，否則不予計分)

1. Prove  $\lim_{x \rightarrow 2} \left(\frac{1}{x}\right) = \frac{1}{2}$  by using  $\varepsilon, \delta$  definition of limit.
2. Find the limit (a)  $\lim_{x \rightarrow \infty} (\sqrt{x+\sqrt{x}} - \sqrt{x})$  (b)  $\lim_{x \rightarrow \pi/2} \frac{\sec(x) - \tan(x)}{x - \pi/2}$  (5,5 分)
3. Suppose  $f(0) = 0, f'(0) = 1$  and  $f(a+b) = f(a) + f(b) + 4ab$ , If  $f'(2)$  exist.  
Find  $f'(2)$ .
4. Find the limit  $\lim_{n \rightarrow \infty} \frac{2}{n} (\ln(n+2) + \ln(n+4) + \dots + \ln(n+2n) - n \ln(n))$
5. Let (a)  $y = x^x$  (b)  $\cos(x+y) = x^2 + xy - \sqrt{y}$  find  $\frac{dy}{dx}$ .
6. (a) Evaluate  $\int_0^2 y^3 \sqrt{1+y^2} dy$   
(b) Find the derivative of  $g(x)$  if  $g(x) = \int_1^{2^x} \cos(y^2 - 1) dy$  (5,5 分)
7. Evaluate (a)  $\int \sin(7x) \cos(3x) dx$  (b)  $\int \sin(x) e^x dx$  (5,5 分)
8. (a)  $\iint_D e^{-y^2} dA$  where  $D$  is a triangle region with vertices  $(0,0), (0,1), (1,1)$ .  
(b)  $\int_0^2 \int_0^1 x^2 y e^{xy} dx dy$  (5,5 分)
9. (a) Find the Maclaurin series of  $f(x) = e^{x^2}$   
(b) Find  $f^{(10)}(0)$  (7,3 分)
10. Maximize and minimize  $f(x,y) = 4xy$  subject to the constrain  $x^2 + y^2 = 50$