

淡江大學 106 學年度日間部寒假轉學生招生考試試題

系別：數學學系資料科學與數理統計組
三年級

科目：機率與統計學

10-1

考試日期：1月6日(星期六) 第1節

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1. (10%) Suppose that A, B, and C are mutually independent events and that $P(A) = 0.5$, $P(B) = 0.8$, and $P(C) = 0.9$. Find the probabilities that none of the events occurs.

2. (10%) If $E(X^r) = 3^r$, $r = 1, 2, 3, \dots$, find the moment generating function of X and $P(X = 2)$.

3. (10%) Find $E(X)$, $Var(X)$ and $P(X > 5 | X > 2)$ when the moment generating function of X is

$$M(t) = \frac{0.4e^t}{1 - 0.6e^t}, \quad t < -\ln(0.6).$$

4. (10%) Let the random variable X be equal to the number of days that it takes a high-risk driver to have an accident. Assume that X has an exponential distribution. If $P(X < 50) = 0.35$, compute $P(X > 100 | X > 50)$.

5. (20%) The serum zinc level X in micrograms per deciliter for males between ages 17 and 19 has a normal distribution with $\mu = 0$ and $\sigma^2 = 4$.

a) (10%) Compute the conditional probability $P(X > 3.92 | X > 3.29)$.

b) (10%) Find the mean and variance of $W = X^2$.

6. (10%) Let X and Y equal the respective numbers of hours a randomly selected child watches movies or cartoons on TV during a certain month. From experience, it is known that $E(X) = 30$, $E(Y) = 50$, $Var(X) = 52$, $Var(Y) = 64$, and $Cov(X, Y) = 14$. Nine children are selected at random. Let Z equal the total number of hours these nine children watch TV movies or cartoons in the next month. Approximate $P(684 < Z < 792)$.

7. (10%) Let $f(x; \theta) = \theta x^{\theta-1}$, $0 < x < 1$, $0 < \theta < \infty$. Find the maximum likelihood estimator of θ .

8. (10%) Let p equal the proportion of Americans who favor the death penalty. If a random sample of $n = 100$ Americans yielded $y = 20$ who favored the death penalty, find an approximate 95% confidence interval for p .

9. (10%) Assume that the weight of cereal in a "12.6-ounce box" is $N(\mu, 0.04)$. The Food and Drug Association allows only a small percentage of boxes to contain less than 12.6 ounces. We shall test the null hypothesis $H_0: \mu = 13$ against the alternative hypothesis $H_1: \mu < 13$.

a) (3%) Use a random sample of $n = 25$ to define the test statistic and the critical region that has a significance level of $\alpha = 0.025$.

b) (3%) If $\bar{x} = 12.92$, what is your conclusion?

c) (4%) what is the p -value of this test?

Note: use the following information to find your answers for Questions 5, 6, and 9.

$$P(|Z| \leq 1) = 0.6826, P(|Z| \leq 1.645) = 0.90, P(|Z| \leq 1.96) = 0.95 \text{ and } P(|Z| \leq 2) = 0.9544 \text{ if } Z \sim N(0, 1).$$