淡江大學 106 學年度日間部寒假轉學生招生考試試題

9-1

系別：數學學系數學組三年級　　　科目：線性代數

考試日期：1月6日(星期六) 第1節　　　本試題共 7 大題， 1 頁

#務必書寫過計算程，否則不予計分。

1. **Find the volume of the parallelepiped determined by u=(0,0,3), v=(1,1, -3), w=(0,1,1). (10points)**

2. **Let** $A = \begin{bmatrix} 1 & -1 & +0 & +0 \\ 0 & +1 & -1 & +0 \\ 0 & +0 & +1 & -1 \\ a & +b & +c & 1+d \end{bmatrix}$. **Find det(A) . (12 points)**

3. **Let** $P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 3 \end{bmatrix}$. **Show that P is invertible and find** P^{-1} **.(12 points)**

4. **Let A be m$\times$n and B be n$\times$m matrices.**

   Prove that if m<n, then BA is not invertible(不可逆). (10 points)

5. **Let** $A = \begin{bmatrix} 6 & -5 \\ 2 & -1 \end{bmatrix}$.

   (a) Find the characteristic polynomial of A. (6points)

   (b) Find an invertible matrix P such that $P^{-1}AP = D$ **is diagonal. (10 points)**

   (c) Find A^{10}**. (10 point s)**

6. **Let A=** $\begin{bmatrix} 1 & -1 & 0 & 2 \\ 0 & -2 & 2 & 4 \\ 1 & -1 & 0 & 3 \end{bmatrix}$ **be 3$\times$4 matrix. (20 points)**

   (a) Show that AX=Y is consistent for all 3$\times$1 matrix Y.

   (b) Find a basis for the solution space of AX=0.

7. **Let** u_1 **= (1,1) and** u_2 **= (1, -1), and let T :** $R^2 \to R^2$ **be the linear transformation such that** T(u_1)= (1, -2) **and** T(u_2)=(-4, 1)

   Find a formula for T(x, y). (10 points)