

# 淡江大學 105 學年度日間部寒假轉學生招生考試試題

系別：物理學系三年級

科目：應用數學

29-1

考試日期：12月3日(星期六) 第2節

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1. (a) (10%) Find all the roots of the equation

$$z^6 + 1 = 0$$

- (b) (15%) Evaluate the sum

$$\sum_{n=1}^N \sin(nx)$$

2. Consider the matrix

$$A = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 2 \\ 0 & 2 & 2 \end{pmatrix}$$

- (a) (15%) Find the eigenvalues and the corresponding normalized eigenvectors of  $A$ .
- (b) (10%) Use the result in (a) to evaluate the matrix  $e^A$ .
3. Consider the vector field  $\vec{v} = (3xy + x^2)\hat{x} + 2yz\hat{y} + (xz + y^2)\hat{z}$ .
- (a) (15%) Evaluate the path integral  $\oint \vec{v} \cdot d\vec{l}$  along the triangular loop  $(0, 0, 0) \rightarrow (2, 0, 0) \rightarrow (0, 1, 0) \rightarrow (0, 0, 0)$ .
- (b) (10%) Use the Stoke's theorem to evaluate the integral in (a)
4. (25%) Find the general solution of the second order ordinary differential equation

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = \cos x$$