淡江大學 105 學年度日間部寒假轉學生招生考試試題

10 - 1

系別：數學學系數學組三年級　　科目：線 性 代 數

考試日期：12 月 3 日(星期六) 第 1 節　　本試題共 6 大題， 1 頁

1. (15 points) Solve the following system by Gauss elimination:

$$
\begin{aligned}
x_1 + x_2 + x_3 &= 4 \\
2x_1 + 2x_2 + 5x_3 &= 11 \\
4x_1 + 6x_2 + 8x_3 &= 24
\end{aligned}
$$

2. (15 points) Let

$$v_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, v_2 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, v_3 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}.$$

Whether vectors v_1, v_2, and v_3 span the vector space $\mathbb{R}^3$?

3. Let A be a 3×3 real matrix given by

$$\begin{pmatrix} -2 & 0 & 6 \\ -1 & 1 & 2 \\ -2 & 0 & 5 \end{pmatrix}$$

   (a) (10 points) Find all eigenvalues and their corresponding eigenvactors of A.

   (b) (10 points) Is A diagonalizable? If yes, find an invertible matrix P and a diagonal matrix D such that $D = P^{-1}AP$.

   (c) (5 points) Find the general form of A^n for $n \in \mathbb{N}$.

4. (15 points) Show that all eigenvalues of a symmetric matrix are real numbers.

5. (15 points) Let A be a symmetric matrix and $\lambda_1 \neq \lambda_2$ be two distinct eigenvalues with corresponding eigenvectors x_1 and x_2. Show that x_1 and x_2 are orthogonal.

6. (15 points) Let $C[0,1]$ be the space of continuous functions on the interval [0,1] with the usual function addition and scalar multiplication, and (standard) inner product given by

$$< f, g >= \int_0^1 f(x)g(x)dx.$$

Let $V = P_2 = span\{1, x, x^2\}$ and apply the Gram-Schmidt algorithm to the basis 1, x, x^2 to obtain an orthogonal basis for the space of quadratic polynomials.