

淡江大學 106 學年度日間部轉學生招生考試試題

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系別：工組三年級

科目：工程數學

考試日期：7月21日(星期五) 第2節

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1. (25%) Suppose $L\{f(t)\} = F(s) = \frac{1}{s^4}$. Consider $L\{f(4t)\} = \frac{4^n}{s^m}$. What are the m and n ?

Note: A table for some functions $f(t)$ and their Laplace Transforms $\mathcal{L}(f)$.

	$f(t)$	$\mathcal{L}(f)$		$f(t)$	$\mathcal{L}(f)$
1	1	$1/s$	7	$\cos \omega t$	$\frac{s}{s^2 + \omega^2}$
2	t	$1/s^2$	8	$\sin \omega t$	$\frac{\omega}{s^2 + \omega^2}$
3	t^2	$2!/s^3$	9	$\cosh at$	$\frac{s}{s^2 - a^2}$
4	t^n ($n = 0, 1, \dots$)	$\frac{n!}{s^{n+1}}$	10	$\sinh at$	$\frac{a}{s^2 - a^2}$
5	t^a (a positive)	$\frac{\Gamma(a+1)}{s^{a+1}}$	11	$e^{at} \cos \omega t$	$\frac{s-a}{(s-a)^2 + \omega^2}$
6	e^{at}	$\frac{1}{s-a}$	12	$e^{at} \sin \omega t$	$\frac{\omega}{(s-a)^2 + \omega^2}$

2. (25%) Please find the general solution for the first-order ODE:

$$\frac{dy}{dx} = \frac{y}{4x} - \frac{x}{4y}$$

3. (25%) Consider a vector field $\mathbf{v}(x, y, z) = v_1(x, y, z)\mathbf{i} + v_2(x, y, z)\mathbf{j} + v_3(x, y, z)\mathbf{k}$, a scalar field

$\phi(x, y, z)$ and the Del operator $\nabla = \frac{\partial}{\partial x}\mathbf{i} + \frac{\partial}{\partial y}\mathbf{j} + \frac{\partial}{\partial z}\mathbf{k}$. Please clearly write down the expressions

of:

- a. Gradient of $\phi(x, y, z)$
- b. Divergence of $\mathbf{v}(x, y, z)$
- c. Curl of $\mathbf{v}(x, y, z)$
- d. Laplacian of $\phi(x, y, z)$
- e. Laplacian of $\mathbf{v}(x, y, z)$

Express your answers in terms of $\phi, v_1, v_1, v_1, x, y, z$.

4. (25%) Please find the eigenvalues and the eigenvectors of the matrix D :

$$D = \begin{bmatrix} 5 & 4 & 2 \\ 4 & 5 & 2 \\ 2 & 2 & 2 \end{bmatrix}$$