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淡江大學 106 學年度日間部轉學生招生考試試題

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系別：數學學系三年級

科目：微積分

考試日期：7 月 21 日(星期五) 第 2 節

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1. (20 分) 判斷下列各式是否收斂，簡述理由。每小題 5 分。

(a) $\lim_{x \rightarrow 0^+} (\sin(x))^x$.

(b) $\sum_{n=1}^{\infty} \frac{1}{n^2 + 1}$.

(c) $\lim_{n \rightarrow \infty} \left(\sin\left(\frac{\pi}{n}\right) \right) \left(\sin\left(\frac{\pi}{n}\right) + \sin\left(\frac{2\pi}{n}\right) + \cdots + \sin\left(\frac{(n-1)\pi}{n}\right) + \sin\left(\frac{n\pi}{n}\right) \right)$.

(d) $\int_0^{\infty} \frac{x}{1+x^4} dx$

2. (40 分) 計算下列各式，每小題 8 分。

(a) Evaluate $\int_0^1 x e^{3x} dx$

(b) Compute $\frac{d}{dx} \int_{x^2}^{e^x} (\tan^2(t^2)) dt$.

(c) Evaluate $\int_0^2 \int_0^1 x^2 y^3 dx dy$

(d) Evaluate $\int_0^1 \int_x^1 \sin(y^2) dy dx$.

(e) Evaluate $\iint_D (x^2 - 4y^2) dA$ where D is the region enclosed by the lines $x - 2y = 1$, $x - 2y = 5$, $x + 2y = 5$ and $x + 2y = 10$.

3. (10 分) Find all local extreme values and saddle points of the function

$$f(x, y) = 3y^2 - 2y^3 - 3x^2 + 6xy.$$

4. (10 分) The plane $x + y + z = 1$ cuts the cylinder $x^2 + y^2 = 1$ in an ellipse. Find the points on the ellipse that lie closest to and farthest from the origin.

5. $a, b \in \mathbb{R}$ and $a < b$. f and g are two continuous functions on $[a, b]$ and $g(x) \geq 0$ for all $x \in [a, b]$.

(a) (5 分) Prove there exists a $c \in (a, b)$ such that

$$\int_a^b f(x)g(x) dx = f(c) \int_a^b g(x) dx.$$

(b) (5 分) Prove that $\lim_{n \rightarrow \infty} \int_n^{n+1} x^2 \cos(x^4) dx = 0$.

6. (10 分) Let $f : \mathbb{R}^3 \rightarrow \mathbb{R}$ be a differentiable function. Prove that f is a continuous function.