

淡江大學 105 學年度日間部寒假轉學生招生考試試題

3-1

系別：物理系光電物理組二年級

科目：微積分

考試日期：12月3日(星期六) 第1節

本試題共 九 大題， 一 頁

\*\*\* 務必寫出計算過程，否則不予計分。 \*\*\*

1. (10%) Find the derivative of the function, if

(1)  $f(x) = (x + \frac{1}{x^2})^{\sqrt{7}}$ .

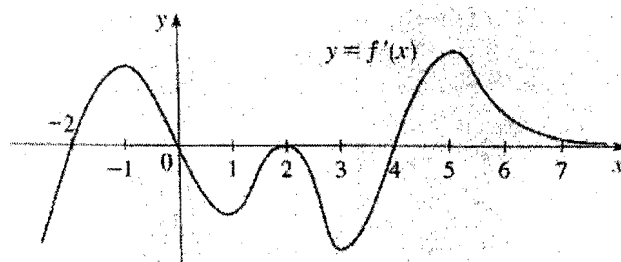
(2)  $f(x) = \int_1^{\cos x} \sqrt[3]{1-t^2} dt$

2. (15%) The following figure shows the graph of the derivative  $f'$  of a function  $f$ .

(1) On what interval is  $f$  increasing?

(2) For what value of  $x$  does  $f$  have a local maximum or minimum?

(3) Sketch the graph of  $f''$ .



3. (5%) Find an equation of the tangent plane to the surface  $z = 3y^2 - 2x^2 + x$  at the point  $(2, -1, -3)$ .

4. (15 %) Find the local maximum and minimum values and saddle points of  $f(x, y) = x^4 + y^4 - 4xy + 1$ .

5. (20%) Evaluate the integral.

(1)  $\int_1^2 x\sqrt{x-1} dx$

(2)  $\int \frac{1}{x^2\sqrt{x^2+4}} dx$

6. (10%) Find the Maclaurin series for the function  $f(x) = \frac{1}{\sqrt{4-x}}$  and its radius of convergence.

7. (10%) Find the points on the sphere  $x^2 + y^2 + z^2 = 4$  that are closest to and farthest from the point  $(3, 1, -1)$ .

8. (5%) Evaluate  $\iint_R y \sin(xy) dA$ , where  $R = [1, 2] \times [0, \pi]$ .

9. (10%) Use polar coordinates to find the volume of the solid that lies under the cone

$z = \sqrt{x^2 + y^2}$  and above the disk  $x^2 + y^2 \leq 4$ .