

淡江大學 106 學年度日間部轉學生招生考試試題

系別：數學學系數學組三年級

科目：線性代數

24-1

3-24

考試日期：7月21日(星期五) 第1節

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6 大題，

2 頁

本試題係由印刷

1. (15 points) Solve the following system by Gauss elimination:

$$x_2 - 3x_3 = -5$$

$$2x_1 + 3x_2 - x_3 = 7$$

$$4x_1 + 5x_2 - 2x_3 = 10$$

2. (15 points) Determine whether the vectors $v_1 = [1, 2, 3, 1, 0]$, $v_2 = [2, 2, 1, 3, 1]$, and $v_3 = [-1, 2, 7, -3, -2]$ in \mathbb{R}^5 are independent.

3. (20 points) Let A be a 3×3 real matrix given by

$$\begin{bmatrix} 4 & 2 & 3 \\ -1 & 1 & -3 \\ 2 & 4 & 9 \end{bmatrix}.$$

(a) Find all eigenvalues and their corresponding eigenvectors of A .

(b) Is A diagonalizable? If yes, find an invertible matrix P and a diagonal matrix D such that $D = P^{-1}AP$.

4. (20 points) Let A be a 4×4 real matrix given by

$$\begin{bmatrix} 2 & -4 & 2 & -2 \\ 2 & -4 & 3 & -4 \\ 4 & -8 & 3 & -2 \\ 0 & 0 & -1 & 2 \end{bmatrix}.$$

(a) Find the rank of A and a basis for the column space of A .

(b) Find a basis for the nullspace $N(A)$. What is the dimension of $N(A)$?

5. (15 points) Determine whether $S = \{1 - x, 2 - 3x^2, x + 2x^2\}$ is a basis for the vector space P_2 of polynomials of degree at most 2.

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6. (15 points) Find all eigenvalues and their corresponding eigenvectors of the linear transformation: $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ defined by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_1 \\ -8x_1 + 4x_2 - 6x_3 \\ 8x_1 + x_2 + 9x_3 \end{bmatrix}.$$