

淡江大學 106 學年度日間部轉學生招生考試試題 2-10

系別：運管系、統計系二年級

科目：微積分

10-1

考試日期：7月20日(星期四) 第2節

本試題共 8 大題， 1 頁

1 (10%) Find the indicated limits if it exists (a)  $\lim_{x \rightarrow 4} \left( \frac{1}{\sqrt{x}} - \frac{1}{2} \right) \left( \frac{1}{x^2 - 16} \right)$  (b)  $\lim_{x \rightarrow -\infty} \frac{\sqrt{x^2 + 9}}{x + 3}$

2. (14%) (a) Find  $\frac{dy}{dx}$  if  $y = (2x + 1)^{\ln x}$  (b) Find  $f'(x) = e^{\sqrt{3x^2 + 1}} - \frac{x}{1 - e^{-x}}$

3. (16%) Determine where the function  $f(x) = x^{2/3} - \frac{1}{5}x^{5/3}$  is increasing or decreasing, and where its graph is concave up and concave down. Find the relative extrema, inflection points.

4. (20%) Find the integral: (a)  $\int_0^1 \int_0^2 x e^{xy} dx dy$  (b)  $\int e^{-2x} / (e^{-x} - 1) dx$

5. (10%) Determine whether the integral  $\int_2^{\infty} \frac{1}{x(\ln x)^2} dx$  is convergent or divergent. Evaluate it if it is convergent.

6. (10%) Find the area between the curves  $y_1 = 12 - 3x^2$  and  $y_2 = 4x + 5$  from  $x = 0$  to  $x = 3$ .

7. (10%) Find all critical points and determine where each corresponds to a relative maximum, a relative minimum, or a saddle points for the function  $f(x, y) = x^3 - y^3 + 6xy$ .

8. (10%) The productivity of a certain company is given by the Cobb-Douglas model as  $P(x, y) = 100x^{1/4}y^{3/4}$  where  $x$  units of labor and  $y$  units of capital are utilized. Each unit of labor costs \$200 and each unit of capital costs \$150. If the company has a total of \$1,600 for labor and capital, how much of each should it use to maximize production?