系別：

1．Consider the linear system $\mathrm{Ax}=\mathrm{b}$ given by $\left[\begin{array}{lll}1 & 0 & 3 \\ 2 & 1 & 5 \\ 4 & 1 & s\end{array}\right]\left[\begin{array}{l}x_{1} \\ x_{2} \\ x_{3}\end{array}\right]=\left[\begin{array}{l}5 \\ 8 \\ t\end{array}\right]$ ．If the matrix $A$ is not invertible，what is the value of s？Apply this value of $s$ and find the value of $t$ that makes the linear system $\mathrm{Ax}=\mathrm{b}$ have a solution．（25\％）

2．Please determine all the eigenvalues and eigenvectors of the following $3 \times 3$ matrix A．（ $25 \%$ ）

$$
\left[\begin{array}{ccc}
2 & -1 & 0 \\
-1 & 1 & -1 \\
0 & -1 & 2
\end{array}\right]
$$

3．If $\mathrm{P}=\left[\begin{array}{cccc}1 & 2 & 3 & 4 \\ 8 & 7 & 6 & 5 \\ 9 & 10 & 11 & 12 \\ 16 & 15 & 14 & 13\end{array}\right]$ ，then $\operatorname{rank}(P)=$ ？$\quad(25 \%)$

4．$(25 \%)$ Solve the following differential equation．

$$
\frac{d y}{d x}-2 x+1=0
$$

