

# 淡江大學 106 學年度碩士班招生考試試題

36 -

系別：數學學系 B 組

科目：統計學

考試日期：3 月 4 日(星期六) 第 2 節

本試題共 6 大題， 1 頁

1. (15%) (a) State the Central Limit Theorem(CLT). (b) Given a pdf  $f(x) = x^3 / 4, 0 < x < 2$ . Find the mean and variance of this distribution. (c) Let  $\bar{X}$  be the mean of a random sample of size 24 from  $f$  in (b), use CLT to approximate  $P(1.4 < \bar{X} < 2)$ . [use the cdf  $\phi(z)$  of  $N(0,1)$  to answer (c)]
2. (25%) Let  $X_1, \dots, X_n$  be a random sample from the pdf  $f(x) = \theta x^{\theta-1}, 0 < x < 1, \theta > 0$ . (a) Find the moment estimator of  $\theta$ . (b) Find the maximum likelihood estimator of  $\theta$ . (c) State the unbiasedness and the consistency of an estimator for the parameter  $\theta$ . (d) Is the estimator in (a) unbiased for  $\theta$ ? Why? (e) Is the estimator in (b) consistent for  $\theta$ ? Why?
3. (15%) Let  $X_1, \dots, X_{25}$  be a random sample from  $N(\theta, 1)$ . (a) State the sufficiency of an estimator of  $\theta$ . (b) Find a sufficient statistic of  $\theta$ . (c) Find a 95% confidence interval for  $\theta$ .
4. (15%) Let  $X_1, \dots, X_9$  be a random sample from  $N(\theta, 1)$ . For testing  $H_0 : \theta = -1$  versus  $H_1 : \theta = 1$ , given a test: reject  $H_0$  if  $\bar{x} = (x_1 + \dots + x_9) / 9 > 0.03$ . (a) Find the significance level of this test. (b) Find the power function for this test? (c) Find the p-value if we have  $\bar{x} = 0.05$ . [use the cdf  $\phi(z)$  of  $N(0,1)$  to answer (a)-(c)]
5. (10%) Given the grades (A-D levels) of 20 students at random from each groups. Use the contingency table to test  $H_0$  : the grade distributions of two groups are the same (with significance level 0.05).

$(\chi_{0.05}^2(2)=6.0, \chi_{0.05}^2(3)=7.8, \chi_{0.05}^2(4)=9.5, \chi_{0.05}^2(6)=12.6, \chi_{0.05}^2(7)=14.1, \chi_{0.05}^2(8)=15.5)$

	A	B	C	D	totals
Group I	2	4	4	10	20
Group II	8	6	6	0	20

6. (20%) Given the independent variable  $x$  and dependent variable  $y$ . The binary variable  $z$  takes 1 if  $y \geq 80$  and 0 otherwise. (a) How to perform a simple linear regression on  $(x, y)$  data? (b) How to perform a logistic regression on  $(x, z)$  data?

$x$	70	74	72	68	58	54	82	64	80	60
$y$	76	84	88	82	70	76	88	78	90	68
$z$	0	1	1	1	0	0	1	1	1	0