淡江大學 106 學年度碩士班招生考試試題

35-1

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系別:數學學系A組 科目:線性代數

考試日期:3月4日(星期六) 第2節

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2 頁

1. (20 pts) True or False. Explain the reason for your answer briefly. (a) Let $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 7 \\ 3 & 7 & 9 \end{bmatrix}$. Then A is diagonalizable.

(b) Let P be an invertible 3×3 matrix over \mathbb{R} . Then $P\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix} P^{-1}$ is invertible.

(c) Let
$$\beta = \left\{ \begin{pmatrix} 1\\2\\3 \end{pmatrix}, \begin{pmatrix} 0\\1\\2 \end{pmatrix}, \begin{pmatrix} 0\\2\\5 \end{pmatrix} \right\}$$
. Then β is a basis for \mathbb{R}^3 .

(d) Let A and B be two 3×3 matrices. Suppose A and B have the same characteristic polynomial. Then A is diagonalizable if and only if B is diagonalizable.

2. (10 pts) We can define an inner product on \mathbb{R}^3 by

$$\langle x, y \rangle = \sum_{i=1}^{3} x_i y_i$$
, for all $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ and $y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$ in \mathbb{R}^3 .

Use Gram-Schmidt process to construct an orthonormal basis γ from the basis

$$\beta = \{ (-1, 0, 1)^t, (1, 1, 0)^t, (1, 1, -1)^t \}.$$

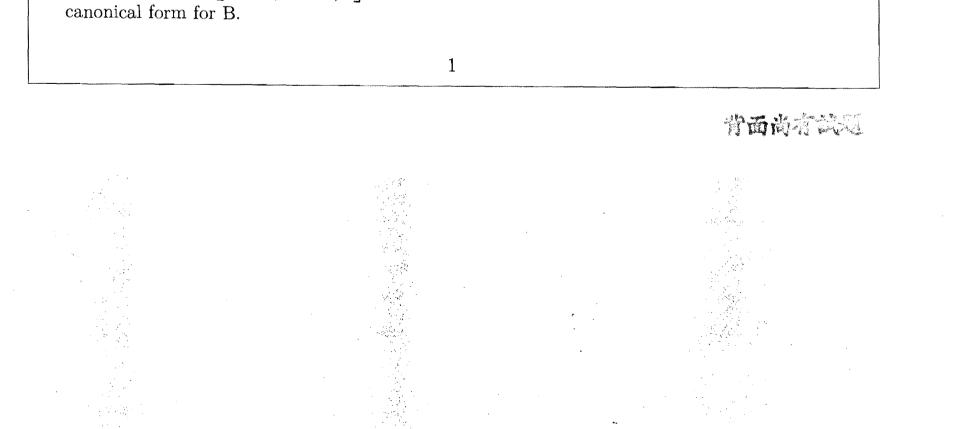
3. Let $A = \begin{bmatrix} \frac{3}{4} & 0 & -\frac{5}{4} \\ \frac{3}{2} & -1 & -\frac{3}{2} \\ -\frac{5}{4} & 0 & \frac{3}{4} \end{bmatrix}$. Define $L_A : \mathbb{R}^3 \to \mathbb{R}^3$ by $L_A(v) = Av$, for all $v \in \mathbb{R}^3$.

(a) (10pts) Find all eigenvalues and eigenvectors of L_A .

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(b) (5pt s) Find a matrix P so that $D = P^{-1}AP$ is diagonal matrix. What is D?

4. (15 pts) Let
$$B = \begin{bmatrix} 1 & 1/2 & -1/2 \\ 0 & 3/2 & -1/2 \\ 2 & -1/2 & -1/2 \end{bmatrix}$$
. Find matrices Q and J so that $J = Q^{-1}BQ$ is the Jordan



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5. (20 pts) Denote V a vector space of finite dimension over \mathbb{R} and $T: V \to V$ a linear transformation. Prove the following statements are equivalent.

- (a) T(v) = 0 for $v \in V$ if and only if v = 0.
- (b) *T* is 1-1.

(c) $\beta = \{v_1, v_2, \dots, v_n\}$ is a basis for V, then $\{T(v_1), T(v_2), \dots, T(v_n)\}$ is a basis for V.

6. (20 pts) Let A be an $n \times n$ real symmetric matrix. Show that $\max_{x \in \mathbb{R}^n, x \neq 0} \frac{\langle Ax, x \rangle}{\langle x, x \rangle}$ is the largest eigenvalue of A and $\min_{x \in \mathbb{R}^n, x \neq 0} \frac{\langle Ax, x \rangle}{\langle x, x \rangle}$ is the smallest eigenvalue of A

