淡江大學 106 學年度碩士班招生考試試題

6-1

系別：數學學系 A 組、B 組　　科目：微積分

考試日期：3 月 4 日(星期六) 第 1 節　　本試題共 10 大題， 1 頁

計算題 (100%，每題 10 分) (需寫出計算過程，否則不予計分)

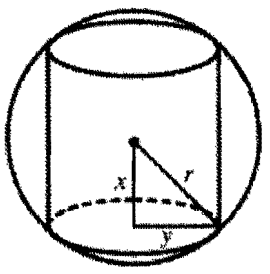
1. Find (a) (5 分) $\lim_{x\to 3}\frac{x^2-9}{x^2+2x-3}$.　　(b) (5 分) $\lim_{x\to -3}\frac{x^2-9}{x^2+2x-3}$.

2. Find (a) (5 分) $\frac{d\,x^x}{d\,x}$.　　(b) (5 分) $\lim_{x\to 0^+} x^x$.

3. (a) (5 分) Find the derivative of the function $g(x)=\int_1^{\cos x}\sqrt[3]{1-t^2}\,dt$.

   (b) (5 分) Find dy/dx if $xy^4+x^2y=x+3y$.

4. A right circular cylinder (圓柱體) is inscribed (內崁) in a sphere (球) of radius (半徑) r. 如下圖. Find the largest possible volume (最大體積) of such a cylinder.



5. (a) (5 分) $\int(x^2+1)(x^3+3x)^4\,dx$　　(b) (5 分) $\int_1^3 x^3\ln x\,dx$

6. Find $\int_0^1\frac{x-1}{x^2+3x+2}dx$

7. (a) (5 分)Determine whether the series $\sum_{n=1}^{\infty}\frac{n(n+2)}{(n+3)^2}$ converges or diverges.

   (b) (5 分)Determine whether the series $\sum_{n=1}^{\infty}\frac{\ln n}{n}$ converges or diverges.

8. Find the Maclaurin series of the function $f(x)=e^x$ and its radius of convergence.

9. (a) (5 分) $\int_1^4\int_0^2(6x^2y-2x)\,dydx$.　　(b) (5 分) $\int_0^1\int_x^1 e^{x/y}\,dydx$.

10. Use polar coordinate to evaluate $\int_{-3}^{3}\int_0^{\sqrt{9-x^2}}\sin(x^2+y^2)\,dydx$.