

系別：數學學系二年級

科目：微 積 分

考試日期：12月3日(星期六) 第1節

本試題共 10 大題， 1 頁

計算題 (每題 10 分，共 100 分) (需寫出計算過程，否則不予計分)

1. Prove  $\lim_{x \rightarrow 3} \left(1 + \frac{1}{3}x\right) = 2$  by using the  $\varepsilon, \delta$  definition of a limit.

2. Let (a)  $\sin(xy) = x^2 - y$ . (b)  $y = (x^2 + x^3)^4$ . Find  $\frac{dy}{dx}$ . (5, 5 分)

3. Find the local and absolute extreme values of the function  $f(x) = x^3 - 6x^2 + 9x + 1$  on the given interval  $[2, 4]$ .

4. (a) Evaluate  $\int_0^2 y^2 \sqrt{1+y^3} dy$ .

(b) Find the derivative of  $g(x)$  if  $g(x) = \int_1^{\cos x} \sqrt[3]{1-t^2} dt$ . (5, 5 分)

5. Evaluate (a)  $\int \frac{1}{2t^2 + 3t + 1} dt$ . (b)  $\int_1^2 x^5 \ln x dx$ . (5, 5 分)

6. Find the volume of the solid obtained by rotating the region bounded by the given curves  $y = 2x, y = x^2$  about the  $x$ -axis.

7. (a) Find the Maclaurin series of  $f(x) = \sin x$ .

(b) Find the interval of convergence of the series  $\sum_{n=1}^{\infty} (-1)^n \frac{x^n}{n^2 5^n}$ . (5, 5 分)

8. Use the Lagrange multipliers to find the maximum and minimum values of  $f(x, y, z) = xyz$  subject to the constraint  $x^2 + y^2 + z^2 = 3$ .

9. Evaluate (a)  $\int_0^1 \int_0^1 ye^{xy} dx dy$ . (b)  $\int_0^1 \int_x^1 \cos(y^2) dy dx$ . (5, 5 分)

10. Use polar coordinate to evaluate  $\int_0^3 \int_{\sqrt{9-x^2}}^{\sqrt{9-x^2}} (x^3 + xy^2) dy dx$ .