淡江大學 105 學年度日間部轉學生招生考試試題

系別：物理學系三年級　　　　科目：應用數學　　　　59-1

考試日期:7 月 22 日(星期五) 第 4 節　　　　本試題共 6 大題，1 頁

1. A matrix is given by $C = \begin{pmatrix} -3 & 2 & 2 \\ 2 & 1 & 3 \\ 2 & 3 & 1 \end{pmatrix}$. (a) Find the eigenvalues and the corresponding eigenvectors of C. (12 pts) (b) Suppose that $f(C) = C^3 - 2C^2 + 5C - 3$, using the results of (a) to evaluate $f(C)$. (8 pts)

2. A vector field is given by $\vec{A} = x^2\hat{i} + y^2\hat{j} + z^2\hat{k}$. (a) Calculate ***directly*** $\oint \vec{A} \cdot d\vec{\sigma}$ over the surface enclosed by a cube with four of its vertices at (0, 0, 0), (0, 0, 1), (0, 1, 0) and (1, 0, 0). (10 pts) (b) Evaluate the same integral by using the divergence theorem. (10 pts)

3. Suppose that $\vec{r}$ is a position vector in Cartesian coordinates and its magnitude is expressed by r. Evaluate (a) $\vec{\nabla}(1/r)$ (6 pts) and (b) $\vec{\nabla} \cdot (\vec{r}/r^3)$ for $\vec{r} \neq 0$. (9 pts)

4. Show that $f(x) = \frac{1}{\pi}\frac{\varepsilon}{x^2 + \varepsilon^2}$ approaches to one-dimensional $\delta(x)$ as $\varepsilon \to 0^+$. (10pts)

5. Solve the following homogeneous ordinary differential equation with degree of three. (15 pts)

$$-3xy^2y' + (2x^3 + y^3) = 0$$

6. (a) Find the Fourier series of the function $f(x) = x$ in the range of $-\pi < x \leq \pi$. (12 pts) (b) Using the results of (a) to evaluate the sum of the infinite series $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \frac{1}{11} + \ldots\ldots$ (8 pts)