淡江大學105學年度日間部轉學生招生考試試題

系別：物理學系三年級　　　　科目：電磁學　　　　40-1

考試日期：7月22日(星期五) 第3節　　　　本試題共 3 大題， 2 頁

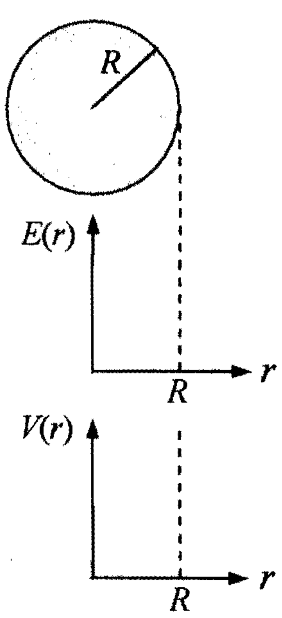
本試題雙面印刷

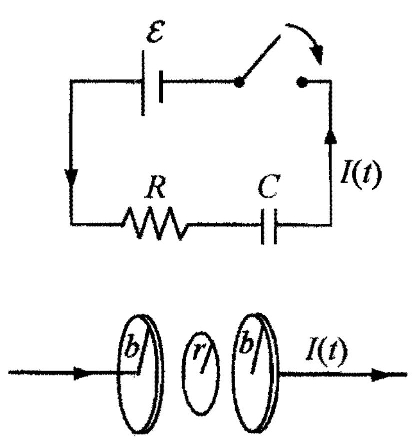
※ 請詳細推導與配置相關圖形，否則不予給分！ ※ 相關公式在第2頁！
※ 請參考下面之圖形！第1題30分；第2題30分；第3題40分！

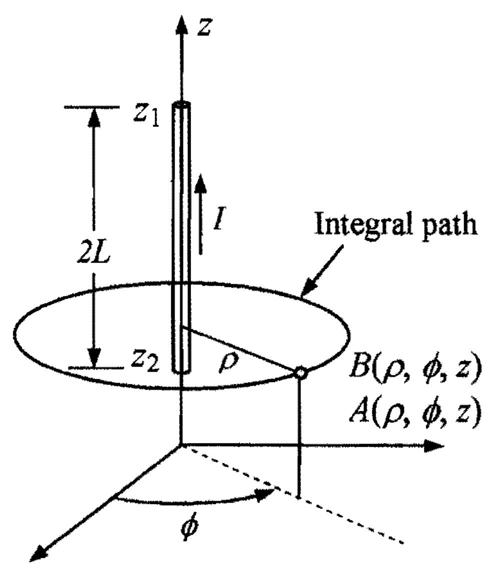
1. A non-conducting sphere of radius R carries a uniformly distributed charge Q.
   (a) Derive and plot the electric field $E(r)$ for $r < R$.
   (b) Derive and plot the electric potential $V(r)$ for $r < R$.
   (c) Calculate $\vec{\nabla}\cdot\vec{E}(\vec{r})$, $\vec{\nabla}\times\vec{E}(\vec{r})$ and $\vec{\nabla}V(\vec{r})$ for $r < R$.

2. At $t = 0$, the switch is closed and we assume that there is no charge on the parallel-plate capacitor. The plates of the capacitor are circular and their radii are b.
   (a) Solve the detailed solution $I(t)$ of the differential equation of a series RC charging circuit.
   (b) Find the final energy stored in the capacitor.
   (c) Find the expression for $\vec{B}(r)$ at a point inside the capacitor at radius r from the center. $(r < b)$

3. The magnetic vector potential of a straight thin wire of length $2L$ carrying steady current I is $\vec{A}(\vec{r}) = \dfrac{\mu_0 I}{4\pi}\ln\left[\dfrac{z_2+\sqrt{z_2^2+\rho^2}}{z_1+\sqrt{z_1^2+\rho^2}}\right]\hat{z}$.
   (a) Set $z_1 = -L$ and $z_2 = L$ and simplify the magnetic vector potential $\vec{A}(\vec{r})$ when $L >> \rho$, that means it is near to the wire.
   (b) Use the result of part (a) to write out the magnetic field $\vec{B}(\vec{r})$ through $\vec{B}(\vec{r}) \equiv \vec{\nabla}\times\vec{A}(\vec{r})$.
   (c) Use the result of part (b) to write out $\vec{\nabla}\cdot\vec{B}(\vec{r})$ and $\vec{\nabla}\times\vec{B}(\vec{r})$.
   (d) Use the result of part (b) to calculate the integral value of $\oint_C \vec{B}\cdot d\vec{\ell}$.







背面尚有試題

※ 相關公式：

$$\vec{\nabla} f = \frac{\partial f}{h_1 \partial q_1}\hat{q}_1 + \frac{\partial f}{h_2 \partial q_2}\hat{q}_2 + \frac{\partial f}{h_3 \partial q_3}\hat{q}_3$$

$$\vec{\nabla}\cdot\vec{A} = \frac{1}{h_1 h_2 h_3}\left[\frac{\partial(A_1 h_2 h_3)}{\partial q_1} + \frac{\partial(A_2 h_3 h_1)}{\partial q_2} + \frac{\partial(A_3 h_1 h_2)}{\partial q_3}\right]$$

$$\vec{\nabla}\times\vec{A} = \frac{1}{h_1 h_2 h_3}\begin{vmatrix} h_1\hat{q}_1 & h_2\hat{q}_2 & h_3\hat{q}_3 \\ \frac{\partial}{\partial q_1} & \frac{\partial}{\partial q_2} & \frac{\partial}{\partial q_3} \\ h_1 A_1 & h_2 A_2 & h_3 A_3 \end{vmatrix}$$