

淡江大學 105 學年度日間部轉學生招生考試試題

系別：數學學系三年級

科目：微積分

38-1

考試日期：7月22日(星期五) 第3節

本試題共 7 大題， 1 頁

第一題為簡答題：共 8 小題，每題 5 分，只需將答案填寫在答案卷上並標明題號，不必書寫過程。

1.

(1) Let $y = \ln\left(\frac{1}{x^3 + 1}\right)$. Find $\frac{dy}{dx}$. (2) Find $\lim_{x \rightarrow 0.5^-} \frac{2x - 1}{|2x^3 - x^2|}$.

(3) Find $\frac{dy}{dx}$ if $y = \int_0^{3x} \sqrt{1+t^2} dt$. (4) Find $\int_0^1 20(1-x)x^3 dx$.

(5) Find an equation of tangent plane to the surface $z = 1 + 2x^2 + y^2$ at the point $(1, 1, 4)$.

(6) Find $\int_0^2 \int_0^1 (3x^2 + 12xy^2) dx dy$.

(7) Find the directional derivative of $f(x, y) = xe^y$ at $P(2, 0)$ in the direction $A = 3i - 4j$.

(8) Find $\lim_{x \rightarrow 0} \left(\frac{e^{2x} - 1}{\sin x}\right)$.

以下為計算題：共 6 題，每題 10 分，務必書寫過計算程，否則不予計分。

2. Find the absolute maximum and absolute minimum values of $f(x) = 2x^3 - 3x^2 - 12x + 1$ on $[-2, 3]$.

3. Let $f(x) = 2x + \cos x$.

(a) Show that f has inverse (證明有反函數). (b) Find $f^{-1}(1)$ and $\frac{df^{-1}}{dx}$ at $x=1$.

4. Find $\frac{dy}{dx}$ at $x=1$ if $y = (x^{3/4} \sqrt{x^2 + 8}) / (3x - 2)^5$.

5. Evaluate the integral $\iint_E y dA$, where E is the region bounded by $y=x-2$ and $x=y^2$.

6. Find the radius of convergence and the interval of convergence of the power series $\sum_{n=0}^{\infty} \frac{2^n x^n}{3n+1}$.

7. If f is differentiable on $(-\infty, \infty)$ and $z = f(x-y)$, show that $\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$.